Search for an Optimal Tonal-System for an Authentic Turkish Soundscape

Weighing several theoretical models on Makam *music against pitch-histograms*

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THE PRESENT-DAY *MAKAM*[1] THEORY SCENERY[2]

In a preceding musicological paper that I had co-authored[3], a groundbreaking analysis was performed in which we juxtaposed 5 contending tone-systems against peaks of collated histograms generated from pitch measurements of renowned masters of Turkish Classical/Art music. The theoretical models of concern were 53-tone Equal Temperament (tET[4]) as used by the Mus2okur software[5]; Yarman-24a as a barebones substitution for the official tone-system[6]; the official Arel-Ezgi-Uzdilek (which is simply a restyling of Rauf Yekta's precursor 24-tone Pythagorean tuning[7]); the derelict Karadeniz-41 that is just a subset of 106-tET[8]; and the contemporary Yavuzoğlu-48[9] that just appropriates – the way I had disclosed back then[10] – Edward J. Hines' 48-tET grid for *makams* – with the first two tunings coming out on top, and the last two acquiring the worst overall rank (*i.e.*, they overshoot or undershoot performed pitches by no less than "a whole comma" in general due to either poor choices or technical hardships in the suitable determination of given *makam* scales).

I recently computed and corrected very minor calculation errors in the mentioned study, to the extent that they do not affect the end results in any significative way. Be that as it may, I will use the updated information when referencing this work hereunder.

Checking the match between said histograms grouped under 9 *makam* categories and several more alternative tunings flooding the market nowadays will reveal whether or not the proposed theoretical solutions actually have true merit in the authentic representation of *makams*[11]. This is especially important since there is

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[1] (Note from the Editors:) The editors are using in this article the Turkish terminology for *maqām* music, with regular plurals when nevertheless in italics. Furthermore, the Turkish ı and i appear identical in the capitalized letters, for consistency with previous publications by NEMO-Online. We have also kept the latin *genus* and *genera* used by the author instead of the Greek *genos* and *genē* used in other NEMO-Online articles.

[2] I am grateful to M. Uğur Keçecioğlu and Anthony Prechtl for their guidance towards my preparation of the cumbersome formulae used in the MS Excel worksheets referred to in this manuscript that facilitated tedious calculations tremendously. I also wish to thank my three exceptional reviewers who commented very positively and constructively. Finally, I thank, in the person of Amine Beyhom, the Editorial team of the NEMO-Online Journal for their painstaking efforts to elevate the quality of this contribution to the highest level. The finalization of this article coincided with the chaotic wake of the massive chemical explosion in Beirut on the 4th of August 2020 concurrent with the second wave of the Covid-19 pandemic; and even so, I benefitted immensely from the professional camaraderie of my colleagues.

[3] [Bozkurt et al., 2009].

[4] Some theorists may also use the abbreviations -ET (standing for "Equal Temperament" or "*Eşit Taksimat*" the way I had Turkicized it), -EDO (standing for "Equal Divisions of the Octave" or "*Eşite Dilimlenmiş Oktav*" the way I had Turkicized it), and -ED2 (standing for "Equal Divisions of 2/1"), with the last one submitted as a terminological "overkill" in driving the point home, if only for the sake of satisfying a methodical concern towards punctiliousness.

[5] [Karaosmanoğlu et al., 2011] – See Appendix B for its encapsulation and adaptation of Arel-Ezgi-Uzdilek pitch ratios.

[6] [Yarman, 2010b, p. 64–99]. (Also *cf.* [Moriarty, 2014] for "MOS" of Ervin Wilson.)

[7] *cf.* [Yarman, 2007a]

[8] [Karadeniz, 1965] and [Yarman, 2007a].

[9] [Yavuzoğlu, 2008].

[10] *cf.* [Hines, 1989] and [Yarman and Karaosmanoğlu, 2009].

[11] If one should ask me why I am using "*maqam*" (Arabic = *maqām*) in one place, and "*makam*" in another place: this is the current tendency when trying to separate the last two centuries of Turkification north of the Levant from the Arabization/Arabicness/Ottoman imperialism of the previous centuries south of Asia Minor. In other

still an ongoing (albeit somewhat abated) crisis with regards to which tone-system is the correct one for traditional music-making in Turkey (and possibly elsewhere, too).

Before I elaborate on what these are, let me first enumerate the digital files that come along with this manuscript:[12]

- *MehmetYektay-65tET-Kanun_Mandallama_Birlesik_Dosya.pdf* (Mehmet Yektay's framework for affixing *mandals* on Turkish *Qanuns* based on dividing the octave into 65 equal parts. He also gives some elemental *makam* scales in degrees of this equal temperament);
- *MehmetYektay-TheoryPractice-65tET-Makamlar.xls* (My effort to decipher and octave-normalize some of Mehmet Yektay's elemental *makam* scales that relate to the scope of this manuscript);
- *Gunalcin-Weiss.xls* (Calculations for the *rast-çargâh* tetrachordal span of Günalçin's "Model 2" versus Weiss's rational[13] "Q9" *Qanun*);
- *Chronicles_Yarman24.xls* (A comparative outline of how the Yarman-24 idea evolved);
- *YA24-to-Mus2.xls* (Schematics of what cent values AEU accidentals get in the *b-c-d* variants of the Yarman-24/31 cast for their incorporation into the Mus2 score editor released by DataSoft);
- *Yarman24d_equalizationOFmeantonefifth.xls* (Sketch of a mathematical optimization with respect to the Yarman-24/31 cast);
- *Yarman24E.pdf* (A monograph in Turkish detailing my Yarman-24/42e venture following in the footsteps of the previous Yarman-24 variants);
- *Yarman36_ahenkler-PB-beats.xls* (Derivation of Yarman-36 from a specially given reference frequency and recommended transpositions);
- *79-tones_mandal-ADO.xls* (String lengths-based *mandal* locations to effectuate the 79-tone *Qanun* tuning);
- *WeighingAgainstHistograms-oldbatch2009.xls* (Original and amended data from the preceding musicological study[14] where theorized *makam* scales had been compared against pitch measurements);
- *WeighingAgainstHistograms-newbatch2020.xls* (Data corresponding to the cornerstone tables and figures of Appendix C in this contribution, where even more theorized *makam* scales are now compared against the same pitch measurements).

The theoretical models I hereby aim to investigate are:

1. *79 MOS 159-tET* formal (with scales "formally defined" in my Expanded Ph.D. Thesis)[15],
2. *79 MOS 159-tET* matching (with closest and most meaningful matches to the peaks instead for the sake of fairness),
3. *Yarman-24a* (with occasional modifications to the original *makam* scales the way I had suggested in the aforementioned [Bozkurt et al., 2009],
4. *Yarman-24b* (that copies the updated *makam* scales of the above),
5. *Yarman-24/31c* (that I lately endorse the most among these Yarman-24 variants [16]),
6. *Yarman-24/31d* (which is an algebraically calibrated version of the above having the same scales[17]),

words, "*Maqam*" – at least to me – is more universal compared to the more recent "*Makam*", which is restricted to the Turkish/Turkic world of especially the past century.

[12] www.ozanyarman.com/files/searchfortheoptimaltonesystem.zip, also available as http://nemo-online.org/wp-content/uploads/2020/10/searchfortheoptimaltonesystem.zip. (This Zip file contains the relevant MS Excel files, featuring two prepared with some essential help from Uğur Keçecioğlu and Anthony Prechtl that greatly facilitated tedious calculations in this manuscript, and which can be freely used for any future study on assessing matches between tunings and histogram peaks.)

[13] Concerning which, arithmetically speaking, *any* interval can be approximated, even to the smallest fraction of a cent, as the ratio of two integers.

[14] [Bozkurt et al., 2009].

[15] [Yarman, 2016, p. 428–462].

[16] [Yarman, 2014b]. This was an invited talk to Istanbul University State Conservatory (7 April, Kadıköy campus) & to Istanbul Technical University Turkish Music State Conservatory-hosted CompMusic Seminar: "Culture specific approaches in music technology" (11 June, Maçka campus). (Also *cf.* "Annotated Bowed Tanbur fretted to Yarman-24 tone-system": https://www.youtube.com/watch?v=bQJVVaVndyg; "Microtones and makam music on TouchKeys": https://www.youtube.com/watch?v=-QcYgslHq9k; "Rast Seyir – Fixed Fret Microtonal Guitar in Yarman-24c": https://www.youtube.com/watch?v=qZXAv8GyHQg.)

[17] [Yarman, 2014c] (Also *cf.* http://www.ozanyarman.com/files/hesap-kitap-hazinesi.zip).

7. *Yarman-24/42e* (that was contrived in an effort to systematize Fikret Karakaya's *perdes*[18])[19],
8. *Yarman-36a* formal (with scales formally defined in terms of our suggested compendium of *genera* with Karaosmanoğlu)[20],
9. *Yarman-36a* matching (with closest and most meaningful matches to the peaks instead for the sake of fairness),
10. *Yektay-65 f*ormal (which Mehmet Yektay formally defined[21] as the "true sixer-*mandal* system" in place of 72-tET on *Qanuns*),
11. *Yektay-65* matching (that is the same 65-tET grid as above – with, however, closest and most meaningful matches to the peaks instead),
12. *Durgun-60* (that is Sait Durgun's ET proposal in place of 72-tET on *Qanuns* – with closest and most meaningful matches to the peaks),
13. *72-tET* (which is the informal resolution on commonplace *Qanuns* – with closest and most meaningful matches to the peaks),
14. *41-tET* (the way I had first brought to attention[22] – with closest and most meaningful matches to the peaks),
15. *34-tET* (same as preceding),
16. *29-tET* (the way I used to consider in line with our apocryphal interpretations of Marchetto di Padua[23] – with closest and most meaningful matches to the peaks).

These are explained in greater detail in Appendix B.

The *makam* categories for this new analysis are once again *RAST* (16 collated samples), *NİHAVEND* (12 collated samples), *KÜRDİLİHİCAZKAR* (17 collated samples), *UŞŞAK* (11 collated samples), *HÜSEYNİ* (15 collated samples), *HİCAZ* (17 collated samples) *SABA* (11 collated samples), *SEGAH* (16 collated samples) and *HÜZZAM* (13 collated samples), constituting a total of 128 audio files,[24] with highly characteristic microtonal inflexions by such celebrated masters as Tanburi Cemil and Yorgo Bacanos.[25]

The obtained results – *i.e.*, both the unweighted and complexity-weighted (and this will be elaborated below) averages across the board – indicate, unsurprisingly enough, the exceptional fidelity of my 79-tone *Qanun* tuning (*i.e.*, "*79-tone Moment of Symmetry out of 159-tone Equal Temperament*", or 79 MOS 159-tET in brief, explained under Appendix B)[26] above all the rest when it comes to pointillistically representing *makams* with a sufficiently high degree of mathematical precision: I call this "PANORAMA A" (See THT 8:100 to THT 17:109 in Appendix C). Though, not quite so much with an additional "efficiency scaling" (and this is elaborated below) as compared to particularly the percentages for 72-tET – as well as for 65-tET and 60-tET to a lesser extent; constituting what I call "PANORAMA B" (THT 18:109 in Appendix C)… Yet, these minutely larger offsets under PANAROMA B in the case of 79 MOS 159-tET are somewhat understandable from the perspective of slightly decreased temperamental regularity and performability.

The obvious downside to such a crowded population of 78 equal + 1 non-equal pitches in 79 MOS 159-tET is hence the challenge in theoretical formalism and difficulty in scale formulation, aside from the taxation of the musician with respect to on-the-fly *mandal* management. Nevertheless, the proposed 79-tone *Qanun* tuning readily compensates for intonational deficiencies because one has a maximum of (as I long since maintained) only 7-8 cents ("c" from this point on) unweighted divergence from any histogram peak – as compared to the as much as "half a comma" deviation of the best contending 60-tET, 65-tET and 72-tET from the same peaks. The consistent happenstance of lowest differences in cents in PANORAMA A for the "peak-matching version of 79 MOS 159-tET" corroborates just this fact; but one may immediately object: "At what cost?"

[18] Fikret Karakaya, who is the founder and director of the BEZMARA Ensemble, communicated his list of traditional Turkish music pitches to this author as extrapolated under *Yarman24E.pdf* in www.ozanyarman.com/files/searchfortheoptimaltonesystem.zip, and as reproduced under Appendix B.

[19] Aforementioned [Yarman, 2014c].

[20] [Yarman and Karaosmanoğlu, 2014].

[21] [Yektay, 2012].

[22] In [Yarman, 2008].

[23] *cf.* [Monzo, 2008] and [Yarman, 2014a].

[24] (Note from the Editors:) The detailed list of the reviewed recordings has been lost, which prevents a thorough verification of the pitch measurements, but we accepted the results on the basis of the acceptance of the 2009 paper [Bozkurt et al., 2009].

[25] [Bozkurt et al., 2009].

[26] *cf.* [Moriarty, 2014].

While the 79-tone *Qanun* tuning's size can be rather unmanageable as such, the "*bulk versus peak-matching performance*" (with this also to be elaborated on further below) of 72-tET, 65-tET and 60-tET – even with their relatively low voluminousness – are still comparable to 79 MOS 159-tET. Even so, their over the top percentages in said measure as shown under PANORAMA B speak more in their favor and against 79 MOS 159-tET. This measure also constitutes a neat way of portraying the tipping point in the balance between a tuning's useful pitch population and its unwieldiness among a group of tunings – as exemplified by the archetypal 72-tET being at or very near said tipping point! Be that as it may, the somewhat better manageability of these three rival resolutions deprives them of overall pitch accuracy when pitted against the 79-tone *Qanun* tuning; so much so that the slight benefits gained do not really seem to be worth the downsizing in this author's opinion.

In contradistinction, any Yarman-24 *a-b-c-d* variant (as a direct replacement for the notorious "Arel-Ezgi-Uzdilek" – or AEU for short – through reliance on the exact same palette of accidentals) does, in spite of being about thrice as sparse, deviate from the measured peak values only by "a comma" at most while performing fairly reasonably. To put things in perspective, the array of efficiency-scaled *and* complexity-weighted grand averages of the mean of maximum differences *or* of the average of differences from histogram peaks (*i.e.*, the so-called "*bulk vs. peak-matching performance*" panorama, or PANORAMA B) is much more satisfactory than for AEU or even its 53-tET encapsulation under *Mus2okur*. The critical question therefore is this: "Why triple the size of the tuning only to gain half a comma finer detail which would still remain vulgar?". Or one may ask in a similar fashion: "Why double the number of conventional tones while not succeeding to overcome the need for commatic alterations in the construction of relatively simple and straightforward *makam* scales?"

At any rate, while 72-tET appears as the definitive benchmark in PANORAMA B – with 65-tET and 60-tET in close pursuit, a nominal Just-Noticeable Difference (JND) of (as promised) 7-8 c limen[27] in the case of 79 MOS 159-tET is arguably much more favorable in terms of authenticity the way I demarcated it in PANORAMA A. Otherwise, if one wishes to secure a modest level of intonational accuracy with minimal damage to *makams* while permitting no excess to accustomed pitches, the lesser-sized tolerable alternatives such as Yarman-24 and Yarman-36 that perform as good as or even better than AEU can be considered. There are evidently even more solutions in-between – such as the lightweight 34-tET and 41-tET also investigated as part of this study,[28] which boast higher moderate degrees of success.

Eventually, the whole ordeal could boil down to my long-standing argumentation that the optimal tuning actually depends on, and is inseparable from, the kind of pitch detail demanded by the *makam* musician; hence my persistence on *not* choosing one single tone-system for *makams* for more than a decade. Is this statement too shocking, after all, when one notices the continuance of the barebones situation with the fret placement on Folk *Bağlamas* as compared to the highly-elaborate yet still unsystematized confounded situation with the long necks of *Tanburs*?

Therefore, if one is hardpressed to establish which tuning is more optimal in the long run for Turkish Classical/Art/Folk music, this can only be soundly achieved through a direct comparison with master pitch histograms generated from the superposition of many audio measurements. Bear in mind that such an excursion yet places no significance on the transposition/modulation or potential polyphonization capabilities of any given theoretical model that might give it an upper edge in the final stage of evaluations. Notwithstanding, nothing less than a quantitative weighing of empirical data will suffice in order to reach a satisfactory conclusion about the *makam*-faithfulness and tractability of a tone-system. Such an up-to-date statistical analysis shall verily be attempted in this manuscript, whereby a FINAL PANORAMA (Table 1:83) serves to showcase the 7 best tunings per size from highest to lowest pitch detail.

The finesse of this study can be improved if (i) the autopeak algorithms are developed further to detect peaks that are visible to the naked eye but anyhow missed during computations, (ii) each *perde* of a given

[27] *cf.* [Long, 2014].

[28] *cf.* aforementioned [Yarman, 2008].

tuning is scaled according to the frequency of occurrence of its autopeak counterpart during weighing and averaging operations, (iii) possible classification errors with given *makams* (*e.g.*, when certain *Hicaz* pieces turn out to be in *Uzzal* instead) and non-stylistic performers (*e.g.*, when early 20th Century non-Turkish musicians get mixed up with 21st Century Turkish musicians) are amended for truer histogram collation.

Methodology

For this paper too, sample collation was based on the automatic tonic frequency identification method developed by Bozkurt[29], whereby the pitch histograms of all the pieces in a certain *makam* were superposed to yield a master pitch histogram. Afterwards, two types of peaks were extracted from these master histograms dubbed "Envelope" and "Average" – against which the corresponding theorized *makam* scales of the tone-systems were pitted. Details can be read in the aforementioned [Bozkurt et al., 2009] wherefrom a synopsis is reproduced in Appendix A.

Given the relative pitches I_{ai} computed from the data automatically and the theorized scale tones I_{ti}, the maximum distance M and the average distance D between the two values for a given *makam* had been calculated as

$$M = \max\{|I_{ai} - I_{ti}|\}, i = 1,2,3..N_x,$$

$$D = \frac{1}{N_x}\sum_{i=1}^{N_x}|I_{ai} - I_{ti}|,$$

where N_x is the total number of scale tones (albeit only being valid within a 2.5 Holderian comma, or HC, vicinity for the older analyzed batch, and within a 2 HC = 45.3 c vicinity for the newer analyzed batch that I consider hereunder) for a given *makam* that correspond to a measured relative pitch.

In addition to the distance, an efficiency measure, E, had been provided, which is the ratio, in percentage, of the number of theorized scale tones within a 2.5 HC (*or, for the newer analysis hereunder, 2 HC*) vicinity of the measured relative pitches, N_x, to the number of tones of the *makam* scale defined in the tuning, N_t.

$$E = (N_x / N_t) \times 100$$

To portray the complexity of a given scale in respect to its tuning, one last measure had been provided: C, which is the ratio, normalized to a percentage, of the number of unused scale tones (*i.e.*, "amount of peak-matching relative pitches" minus the "total number of tones in the tuning"), N_z -N_x, to the total number of tones in the tuning, N_z.

$$C = (1 - N_x / N_z) \times 100$$

Notice, M, D, E and C were calculated for *envelope* and *average* histogram peaks separately – but their means will be brought together at the final stage. Thus, the mean of the entire set of maximum differences and the mean of the entire set of averages of differences will also be computed for each tone-system. Last but not least, further manipulations shall yield complexity and/or efficiency scaled grand averages, as well as one final machination which I call the FINAL PANORAMA that showcases the best players. All these are tabulated in Appendix C.

Scaling via the complexity measure is different in this paper in comparison to the preceding[30] study. Because a direct multiplication by any complexity percentage results in the diminishing of all cent offsets, the solution is to choose the lowest complexity percentage in the list as the basis for the following upscaling operation:

{(*Current complexity percentage*/100) + (1 – [*smallest among the list of complexity percentages*/100])} × *error in c*

The left-hand-side thus gives sensible coefficients for a fair portrayal of complexity-weighted cent errors. When all is said and done, efficiency upscaling can then be introduced to brutally curb down advantageous tunings due to their finer granularity. Since multiplying with any full efficiency would preserve the error while multiplying with half the efficiency would just decrease the error, the correct approach is to divide an input value by the corresponding efficiency percentage so as to arrive at PANORAMA B and then the FINAL PANORAMA.

[29] [Bozkurt, 2008].

[30] [Bozkurt et al., 2009].

Methodological caveats

Now, one may inquire as to what happens when the analyzed pitches constitute vibratos or portamentos. To this I respond as follows… The histogram peaks are already extracted from audio recordings that include such features by master performers; if the inflexions and microtonal ornamentations were amiss, then we would observe narrow spikes. This is yet not an issue with the current contribution. In the case of even more vibratos and portamentos, the autopeaks would ordinarily be less pronounced (their kurtosis would decrease – *i.e.*, the skirts of such elevations would horizontally spread). Since these are all normalized to 1 (*i.e.*, highest amount of occurrence), and because they are the result of collations of many recordings deemed to be in a given *makam*, their heights or positions on the pitch space would not change by much, if at all.

So how do these aspects actually reflect in the computations? Are the contours and shades surrounding the immediately perceived pitch (such as attack culminations or minute inflections of the melodic curve) taken substantially into account? Well, yes! These subtleties of pitch – depending on the style, mannerisms, artistic skill and technical capability – are part of the natural performance by the masters on their instruments that is under scrutiny. The pitch measurement and autopeak generation method aptly takes into consideration this reality of the audio recordings, because any characteristic inflexion, if prevalent enough, already contributes to the rise of an histogram peak or makes a bump in one of the valleys of the histogram. The expected results are then outlined in the previous paragraph.

Yet, I did not quite delineate what would happen if other recordings were included in the search... In reality, it depends. If the analyzed historical epoch and class of artists had conspicuous variance in terms of style, mannerisms or instrumentation; that is to say if a group of Maghribi or Iranian performers were brought into the mix with Turkish performers, then one would most likely expect to see wild results in the histograms – such as with "*perde segâh*" having many distinct peaks in a *makam* that shouldn't have it in the classical Turkish understanding, for instance. Likewise, if *twelvulated polyphonalist executants* of Turkey were included along with traditional Turkish musicians, one might falsely conclude that "*perde segâh*" in – say – *Hüseyni makam* is actually closer to a tempered wholetone above its tonic, for instance.

To do plausible objections in this regard any justice, one has to restrict the sample set to only the class of musicians that autochthonously belong to the genre that is the topic of research. In such a case, while earlier or later periods may reveal some deviations in the peak heights and positions on the pitch space, ordinarily one would not expect it to be by too much (*e.g.*, if "tradition" means anything!)[31]. This is especially so when the sample set includes many collations in given *makams*. As more of the same class of performers and *makams* are thrown into the mix, the result should be similar to what is already presented in [Bozkurt et al., 2009] as well as in the current paper. In other words, with the available dataset, one already may be entitled to speak of a "homogenized autopeak panorama" for the *makams* of concern.

Newly analyzed tunings

Because of the lack of any formally defined *makam* scales under 72-tone Equal Temperament (72-tET), the closest matching degrees to the histogram autopeaks – in just the same way as for 29-tET, 34-tET, 41-tET and 60-tET – were handpicked. This "6-fold detailed 12-tET" resolution is particularly applied to Turkish *Qanuns* due to the prevalent usage in Turkey of electronic tuners imported from abroad of late (the way I

[31] As underwhelming as it may sound, the occasionality (the way pointed out by Amine Beyhom in [Beyhom, 2014]) of the septimal tetrachord 15/14 x 7/6 x 16/15 (as alluded to by Rauf Yekta further down in the text) – in being one continued variant of the *Hicaz genus* across decades (if not centuries) of classical performance – finds its almost perfect counterpart within Yekta-Arel-Ezgi-Uzdilek and naturally its 53-tET encasing, too. When overlaid with 12/11 x 7/6 x 22/21 (also alluded to by Rauf Yekta further down in the text), one might expect either a single median peak, or separate peaks for the second degree of the *Hicaz* scale in a given histogram. Yet, in FHT 11:105 (for "Figure Hors Texte" – or "Plate" – no. 11, p. 105), the auto-peak algorithm is just not sensitive enough to differentiate the minutiae of the second degree of *Hicaz* if there indeed are such *genera* variations. Even so, the location of any tell-tale peaks is not off by too much in just the way required of an established branch of persisting tradition.

had explained right at the onset[32] of my doctorate dissertation[33]– whereby the halftone *mandal* is situated at an equal semitone, and the remaining space to the nut *mandal* is divided into 6 equidistant parts[34]. This resolution, being a multiple of 12-tET, also incorporates 24-tET and 36-tET as rigorous subsets; the former of which is the notorious Arabic quarter-tone scale[35], and the latter of which was (sort of) a preliminary proposal in my master's thesis[36]. Suffice it to say, one may manipulate the pertinent MS Excel spreadsheet[37] to fiddle around with these and other tuning proposals not considered as part of this study.

Coming next, Mehmet Yektay, grandson of the famed theorist Rauf Yekta (1871-1935), sometimes gave a whole-tone or a major-third moved scales without any mention, and sometimes failed to correctly identify the optimal *makam* scale pitches from his 65-tET proposal,[38] leading to conspicuous divergences from the histogram peaks. His original scales are tabulated in another MS Excel document[39] accompanying the one just mentioned. These shortcomings have been compensated herein by taking separately the best available 65-tET degree matches to the measured peaks. Such a high resolution was anyway proposed by Yektay as the "true sixer-*mandal* system"[40] in place of 72-tET following in the Pythagorean footsteps of 53-tET.

A similar situation arose in the case of the "formally defined" *makam* scales under my 79 MOS 159-tET, as well as under Yarman-36; with the best-matching subsets being hence proposed besides for the sake of fairness. Note that 79 MOS 159-tET *makam* scales were all transposed on C4 = 262 Hz because of my trust in the mathematical symmetry and completeness of the system (*cf.* Appendix B). Thence, moving the scales on any other degree should more or less yield the same results to all intents and purposes.

As a side note, the rather unpropitious juxtaposition of the "formal scale pitches" of these tunings with the histogram autopeaks may evince my ineptitude, as a Western Classical music acclimatized Pianist, of recognizing the proper traditional or modern application of *makams* (which happens to be just as bothersome a situation for Yektay and Yavuzoğlu!), or my penchant to capture an even truer authenticity the way I professionally envision (such as, for instance, concerning my ongoing personal belief in the historical *Nihavend* being a 5-limit Just Intonation minor and the historical *Buselik* being instead a characteristically 3-limit Pythagorean minor or even a supraminor).[41]

Owing to the fact that Durgun60 is the same 60-tET resolution as that initially defended by Yavuzoğlu[42] – which he seemingly abandoned during the course of time in favor of 48-tET[43] – and seeing as different *makam* scales can be later extrapolated to add to the confusion, I have once more preferred to adopt only the best correlations with histogram average and envelope peaks.

Moving on, my Yarman-24 series of tunings operate within a maximum of "one comma inflexion margin" for any given pitch just as I have claimed for years,

[32] [Yarman, 2006].

[33] [Yarman, 2016].

[34] *cf.* http://ozanyarman.com/wpress/2013/03/119 under [Yarman et al., 2019]. Also *cf.* Ozan Yarman's presentation at the DR. İBRAHİM ÜZÜMCÜ stage whose video was uploaded to https://youtu.be/_Ro5b8CIEn8 *and* his Power Point slideshow the video of which was uploaded to https://youtu.be/R0DnADsS05I; for a correctly typesetted PDF document, visit http://www.ozanyarman.com/files/DrOz_perde-seyirV14-ENSON.pdf *and* for additional genuine information about Eurogenuous *mandal* placement practically yielding 72-tET on quotidian Turkish *Qanuns*, see particularly http://ozanyarman.com/wpress/2013/03/119.

[35] [Touma, 2003].

[36] [Yarman, 2002, p. 44–56].

[37] Downloadable within www.ozanyarman.com/files/searchfortheoptimaltonesystem.zip.

[38] [Yektay, 2012].

[39] *MehmetYektay-TheoryPractice-65tET-Makamlar.xls.*

[40] Which I happened to contest in a private e-mail to Yektay dated 13 May 2009 for being one of the worst tunings of such great size for approximating simple superpartient and epimoric 7-limit intervals like 7:4, 10:7, 7:5, 9:7 and 7:6 that take place in *Segah*, *Hicaz* and *Saba genera* and scales all over – with respective absolute errors of 8.826 *c*, 8.26 *c*, 8.26 *c*, 7.993 *c* and 8.41 *c*, which are altogether quite unfavorable, inasmuch as adversely impacting, among other things, particularly the *Hicaz* genus in my opinion.

[41] For readers not familiar with the mathematical terminology, *n-limit* denotes the highest prime number (or sometimes just the odd number) obtained through the factorization of both the numerator and denominator of a given ratio or set of ratios in order to demarcate complexity; whereas *Pythagorean* means that the ratio is arrived at through the concatenation of some pure fifths (iterative multiplications by 3:2 or 2:3) or pure fourths (iterative multiplications by 4:3 or 3:4), usually followed by octave normalization.

[42] In [Yavuzoğlu, 1991].

[43] [Yavuzoğlu, 2008], which had been identified in the aforementioned precursor to this study [Bozkurt et al., 2009] as having the lowest overall rank among the five competing tone-system candidates.

and as evidenced by the data, too. In any case, all Yarman-24 variants perform comparably better than AEU. To keep track of which variant was brought forward and when, another MS Excel document named above as *Chronicles_Yarman24.xls* is included in the aforementioned complementary ZIP file. Appendix B also provides additional information in this regard.

Again, be mindful that the results of this paper say nothing about the transposition/modulation or prospective polyphonization capabilities of any given tone-system, or, for that matter, unique bonus features such as the 12-tone cyclic subset or AEU imitation subset availability in the case of my 79-tone *Qanun* tuning the way I particularize under Appendix B. Additional measures may need to be devised to weigh such capabilities against a theoretical model's ultimate representational success.

Numbers are in Appendix C. Rast scale[44] distributions of the theoretical models contra autopeaks are given in THT 8:100[45], and mismatches against 16 collated pitch-histograms of *RAST* are plotted in FHT 6:100[46]; *Nihavend* scale distributions of the theoretical models contra autopeaks are given in THT 9:101, and mismatches against 12 collated pitch-histograms of *NIHAVEND* are plotted in FHT 7:101; *Kürdilihicazkar* scale distributions of the theoretical models contra autopeaks are given in THT 10:102, and mismatches against 17 collated pitch-histograms of *KÜRDILI-HICAZKAR* are plotted in FHT 8:102; *Uşşak* scale distributions of the theoretical models contra autopeaks are given in THT 11:103, and mismatches against 11 collated pitch-histograms of *UŞŞAK* are plotted in FHT 9:103; *Hüseyni* scale distributions of the theoretical models contra autopeaks are given in THT 12:104, and mismatches against 15 collated pitch-histograms of *HÜSEYNI* are plotted in FHT 10:104; *Hicaz* scale distributions of the theoretical models contra autopeaks are given in THT 13:105, and mismatches against 17 collated pitch-histograms of *HİCAZ* are plotted in FHT 11:105; *Saba* scale distributions of the theoretical models contra autopeaks are given in THT 14:106, and mismatches against 11 collated pitch-histograms of *SABA* are plotted in FHT 12:106; *Segah* scale distributions of the theoretical models contra autopeaks are given in THT 15:107 and mismatches against 16 collated pitch-histograms of *SEGAH* are plotted in FHT 13:107; lastly, *Hüzzam* scale distributions of the theoretical models contra autopeaks are given in THT 16:108, and mismatches against 13 collated pitch-histograms of *HÜZZAM* are plotted in FHT 14:108.

"PANORAMA A" and "PANORAMA B" grand averages are presented in THT 17 and THT 18:109 respectively. Table 1:83[47] features the "FINAL PANORAMA" where 7 different tone-systems especially rise to the forefront.

CODE AND DATA

The MS Excel spreadsheet document named as *WeighingAgainstHistograms-newbatch2020.xls* referred to in this study[48] employs specially crafted formulas to facilitate the computation of the rightmost values (the last four columns) throughout Tables 1-9 after a simple copy-pasting of tabulated quantities; wherefrom the rest of the averaging and complexity and/or efficiency weighing can be accomplished straightforwardly. To begin with, in the *HÜZZAM* sheet for instance, the formula in cell C24

```
=IF(D4>0,IF(ABS(C4-C$19)<ABS(D4-C$19), ABS(C4-C$19),ABS(D4-C$19)),ABS(C4-C$19))
```

automatically looks at two neighboring cells containing the scale tones for "*perde çargâh*" and picks the one that is the closest match to the histogram autopeak value so as to take its absolute difference from the peak's cent. All the way to the end of the row under consideration operates in the same fashion until one stumbles upon the end of the table, which necessitates the formula in cell Q24

```
=IF(Q$19<>P$19,ABS(Q4-Q$19), IF(P24=0,ABS(Q4-Q$19),0))
```

that conditionally computes the absolute difference between the scale tone of the last column with the nearest

[44] Here, lowercase names refer to *makam* scales only. Sometimes I fully capitalize the *makam* name, as I have done here, for visual emphasis. Otherwise, when the *makam* name is entirely uncapitalized, including the first letter, that denotes a synonymous *perde* name instead.

[45] For "Tableau Hors Texte" no. 8, p. 100.

[46] For "Figure Hors Texte" – or "Plate" – no. 6, p. 100.

[47] Which is inserted at the end of the discussion of the results in the main text.

[48] See footnote 37:79.

of the two final neighboring autopeak values in said row.

After the differences are calculated as such, the expression

```
=SUMPRODUCT(MAX((C24:Z24<45.3)*C24:Z24))
```

finds the maximum difference lower than 45.3 *c* (*2 Holderian commas*) throughout the given row, and

```
=SUMIF(C24:Z24,"<45.3",C24:Z24) /
ROUND(SUMPRODUCT(--(C4:Z4<>"") *
(C$19:Z$19<>"") /
COUNTIF(C$19:Z$19,C$19:Z$19&"")),0)
```

returns the average of the differences in said row based on the number of scale tone matches to the histogram peaks as the divisor. The procedure up to this point is then repeated for the other type of autopeak.

Once the formula bit for the number of scale tone matches to the number of peaks

```
=ROUND(SUMPRODUCT(--(C4:Z4<>"") *
(C$19:Z$19<>"") / COUNTIF
(C$19:Z$20,C$19:Z$19&"")),0)
```

is divided by another formula, *i.e.*,

```
=SUMPRODUCT((C4:Z4<>"") /
COUNTIF(C4:Z4,C4:Z4&""))
```

that establishes the total number of suggested tones in the theoretical model for that *makam*, we get the efficiency measure – which can be multiplied by 100 to display it in percentage.

For the complexity measure, remember that one multiplies by 100 the ratio of the unused scale tones to the total number of tones in the tuning. The second longest and rounded formula bit above already gives the number of unused tones if we subtract the known number of pitches per octave from it (in this case, 79-9=70) – wherefore, dividing this last part by the known number of pitches per octave and multiplying by 100 yields the complexity (*e.g.*, [70/79]*100=88.6%).

Proper scaling by this complexity was already explained under the Methodology section. Additional upscaling by efficiency is simply done via dividing the complexity-weighted result by the efficiency percentage value as previously mentioned.

And that's basically it.

As stated in the previous section, the treasure trove of data computed by these formulae is reproduced below in Appendix C.

I may now proceed to discussing them in the next and final section.

Discussion of results

Going forward on a *makam*-by-*makam* basis across all the Tables and Figures (*cf.* Appendix C), we see that 79 MOS 159-tET (hereinafter the "peak-matching version") is among the few models that represents *RAST* best when the divergences are unweighted (*cf.* THT 8:100 & FHT 6:100). This, and the case with other *makams* also, is on par with my more than a decade long perseverance to not force upon people what *makam* scales should be.

When divergences are complexity-weighted, though, Yarman-24b comes out on top; here the proximity of Yarman-24b and Yarman-24/31c to *RAST* is quite noticeable (*cf.* THT 8:100 & FHT 6:100).

For *NIHAVEND*, 72-tET and 79 MOS 159-tET just surpass Sait Durgun's 60-tET, insofar as sharing the pedestal when the divergences are unweighted. The situation is similar when they are complexity-weighted, too (*cf.* FHT 7:101 & FHT 7:101).

Same with *KÜRDILIHICAZKAR*; save that 79 MOS 159-tET excels over the rest when the divergences are complexity-weighted (*cf.* THT 10:102 & FHT 8:102).

Unweighted *UŞŞAK* is still better represented by 79 MOS 159-tET, and complexity-weighted UŞŞAK is much more neatly represented by 72-tET (*cf.* THT 11:103 & FHT 9:103). Of particular note for this *makam* are Yarman-24/42e's comparatively small average deviations from the peaks.

Sait Durgun's 60-tET is the winner for *HÜSEYNI* in both venues (*cf.* THT 12:104 & FHT 10:104), with 79 MOS 159-tET followed by 72-tET and 65-tET in close pursuit.

79 MOS 159-tET is the winner for *HICAZ* in both venues (*cf.* THT 13:105 & FHT 11:105), while 41-tET competes admirably.

SABA too is the undisputed domain of 79 MOS 159-tET in both venues (*cf.* THT 14:106 & FHT 12:106), whereas 65-tET, 72-tET, 41-tET, 34-tET and even 29-tET in turn exhibit a worthwhile credibility for this *makam*.

60-tET and 72-tET compete for dominance over *SEGAH* in both venues (*cf*. THT 15:107 & FHT 13:107), with 79 MOS 159-tET and 65-tET and even Yarman-24/42e just barely shy of them.

Mehmet Yektay's 65-tET is the "king of *Hüzzam*" in both venues (*cf*. THT 16:108 & FHT 14:108). Be that as it may, the 79-tone tuning, 72-tET, 60-tET and Yarman-24/42e are perceptibly runners-up; with 72-tET and 60-tET overshadowing the rest (except 65-tET) when the divergences are complexity-weighted.

In THT 17:109, where the 79-tone *Qanun* tuning is the penultimate champion, PANORAMA A shows how intelligently increasing the number of pitches in an octave can expedite a well-crafted temperament to outclass small contenders as well as to overshadow rival biggies. The situation with 72-tET, 65-tET and 60-tET coming in second place demonstrates this clearly.

Yet, PANORAMA B in THT 18:109 communicates instead that, if one keeps increasing pitches after a certain point, it will not be in that tone-system's favor anymore. In other words, for the present study, one comes to settle on 72-tET as the benchmark at a score of 100% with regards to the entirety of efficiency-upscaled grand averages of weighted maximums and differences: It boasts the lowest upscaled cent values across the board – with 65-tET and 60-tET just on its heels.

Here, out of the entire list of given tunings, 72-tET thence seems to be more or less the tipping point (*e.g.*, *bulk vs. peak-matching performance benchmark*) before things start to get worse for ever finer divisions of the octave. Among the given list, too few pitches don't seem to work well in terms of *makam*-fidelity either.

This panorama also reveals that music theorists bungle a lot when trying to impose their formal scales on traditional and modal art forms. Just look at the formalisms of particularly Karadeniz, Yektay and Yavuzoğlu…[49] Furthermore, the mediocre performances of several cherished middle-ground tunings such as 53-tET and Yarman-36 are indeed disappointing.

To recapitulate, I wish to draw attention to the FINAL PANORAMA in Table 12, where one may especially notice how Yarman-24/42e falls behind 34-tET of an equivalent milieu, and Yarman-24/31c falls behind 29-tET of the same league, and how *Mus2okur*'s 53-tET cannot compete against the earliest Yarman-24 variants ("a" and "b") less than half its size. Once again, this panorama either demonstrates unforgivable blunders of music theorists in their *makam* scale formalism, or hints at an insidious shift in the modern Turkish intonation soundscape in accordance with what I some years ago christened as *temperialism*: "Conscious or inadvertent equal temperament infusion through cultural imperialism".[50]

On the other hand, I did not anticipate how useless a pursuit Yarman-24/42e would prove to be with respect to the ever-so-delicate prevalence of 41-tET over it. Apparently, even with considerable transpositional sacrifices resulting in much irregularity, one should find it very hard to compete against suitable equal temperaments of identical caliber.

[49] This incidentally reminds me of a couple of thought-provoking past comments by my detractors that goes like: "This *Qanun* has no educational significance!" or "First you will learn Turkish music and only then speak on Turkish music!" or "…Ekrem Karadeniz, who is the only other system-possessing music theorist!", etc…

[50] I suspect the generation preceding – but also including the prime years of – Necdet Yaşar, Niyazi Sayın and Nevzat Sümer had little to no qualms about what I refer to as "temperialism", as evidenced by modern pitch measurements showing how these venerable masters more or less upheld tradition. It likely affects the succeeding generations that are more susceptible to the electro-mechanized society norms of highly advanced technological development coupled with the decadence of the masses, and especially those who must work alongside popular music groups in studios, concert venues, and public spaces. Even so, *caveat emptor*: In contrast to the befuddled "orthodoxy nostalgia" or "tradition romanticism" the way exemplified by Okan Murat Öztürk (*cf*. [Öztürk, 2019a ; 2019b] – against whom my recent criticisms in the comments section of the given websites went unanswered by the way – a pioneering study by Amine Beyhom [2014] hints at the necessity to question what "*makam* tradition" should actually stand for in the face of Beyhom's disclosure of a detectable intonation shift in the *makam* performances by the aforementioned masters throughout a 30-year span. While "*Arelization*" more than "*temperialism*" could have played a role in such a shift, still, if one is to talk of an observant branch of praxis throughout the past century despite all political and theoretical revolutions and setbacks, one might perhaps do well to consider such revealed changes as part of a more general "oscillation" or "perturbation" belonging to the "body of tradition" itself (*e.g.*, "*Hafızlık Ekolü/Tavrı*" *vs.* "*Fasıl*" *vs.* "*Aşıklık Geleneği*" – *cf*. especially [Beyhom, 2019]).

What was rather unexpected for me was the slight worsening in discrepancies under Yarman-24/31c (*cf*. PANORAMA A under THT 17:109) as compared to the earlier Yarman-24 variants. While I knew that the "d" variant did not resonate so well digitally, I still expected the "c" variant to be an improvement over the "a" and "b" variants. Could it be that the overpowering climate of *temperialism* is adversely affecting this author also?

Much more can be said about the numbers in the FINAL PANORAMA as well as PANORAMA B. What I found particularly surprising is how 34-tET performed better than 41-tET on the whole or even singularly with respect to some "crunchy"[51] *makams* like *Uşşak* and *Saba*.

But may the reader look at just how good 29-tET performs in comparison to AEU or even its 53-tET encapsulation under Mus2! Such an outcome is definitely surprising. In addition to these, PANORAMA A divergences for 29-tET and 53-tET are very much comparable, too (*cf*. THT 17:109).

With respect to the Yarman-24 variants, the "near-Pythagorean" 29 equal division of the octave does not require a notational paradigm shift either. In other words, the habitual AEU accidentals symbolism can be applied to it right away.

One then only wonders why this resolution was not chosen a century ago as the barebones container of *makams*. Given the current theoretical mess, it seems a pity indeed...

The results especially reflect how there is still no well-performing middle-ground division of the octave between the resolutions of 34 pitches to the octave and 60 pitches to the octave. This is significant if one wishes to compete against the bulwark success of voluminous tone-systems surpassing 53-tET.

Ultimately, the FINAL PANORAMA (Table 1) can be said to reveal the predominance of (1) 79 MOS 159-tET over all else, followed by (2) 72-tET, followed by (3) 65-tET, followed by (4) 60-tET, followed by (5) 34-tET, followed by (6) 29-tET, and at last followed by (7) Yarman-24a.

FINAL PANORAMA	Final Average of maximums		Final Average of mean diff.	
79 tone-formal (79 per 159 tET)	31.7	35%	13.5	42%
79 tone-matching **(nr.1)**	**11.2**	**100%**	**5.8**	**98%**
Yarman-24a **(nr.7)**	**18**	**62%**	**7.2**	**79%**
Yarman-24b	19.5	58%	7.5	76%
Yarman24/31c	20.8	54%	7.5	75%
Yarman24/31d	20.9	54%	7.8	73%
Yarman24/42e	15.5	72%	7.8	72%
Yarman36a-formal	23.9	47%	12.4	46%
Yarman36a-matching	23	49%	9.7	58%
Yektay65-formal (in 65 tET)	27.6	41%	11	51%
65 tET-matching **(nr.3)**	**12.2**	**92%**	**5.8**	**97%**
60 tET-matching **(nr.4)**	**12.3**	**91%**	**6.2**	**91%**
72 tET-matching **(nr.2)**	**11.8**	**95%**	**5.7**	**100%**
41 tET-matching	15.6	72%	7.5	75%
34 tET-matching **(nr.5)**	**15.6**	**72%**	**7.3**	**77%**
29 tET-matching **(nr.6)**	**16.9**	**67%**	**8**	**70%**
YAEU (24 per 53-tET)	24.5	46%	9.1	62%
Mus2 (53 tET-formal)	23.3	48%	10.1	56%
Töre-Karadeniz (41 per 106 tET)	30.5	37%	12	47%
Old Yarman-24a	19.6	57%	7.4	77%
Yavuzoğlu-48 (in 48 tET)	28.9	39%	14.2	40%

Table 1 FINAL PANORAMA – with the bottom 5 rows drawn from [Bozkurt et al., 2009] – where the mean of M_c, GWM and the mean of CEM, GEM are averaged to yield *final average of maximums*, while the mean of D_c, GWD and the mean of CED, GED are averaged to yield *final average of mean differences*.[52]

* * *

[51] What I mean by it is "*intonation-wise sticking out by liberally bent perdes*". In *SABA* and *HICAZ* in particular, whose characteristically variegated supple *perde* corresponds to the *makam*'s name (for instance, *SABA* gets its name from "*perde saba*" or vice versa – which, on the *Ney*, is the same hole for "*perde hicaz*" of *HICAZ*), one may see Maghribis play it "semitonally" above *perde çargâh* in both cases, and Turks as well as Iranians "sesqui-semitonally" in *SABA* and "semitonally" or even "quarter-tonally" in *HICAZ* – forming thus a justly intoned major third versus either an acute major third for *SABA* or a neutral third for *GARIP HICAZ* above the tonic respectively.

[52] Please refer to the explanations in the captions to THT 17 & THT 18:109 for M_c, GWM, CEM, GEM, D_c, GWD, CED and GED. Likewise to the procedure explained in the caption to THT 18:109, percentages are found according to the formula (*smallest value in column two or four*) / (*current value in column two or four*). This panorama highlights 7 tunings above the rest with 79 MOS 159-tET (peak-matching version) staying in the lead once more. Best values are highlighted and in bold, while worst values are striped in red. Smaller values corresponding to greater percentages are always better.

Appendix A

A summary of the frequency analysis procedure used in this research

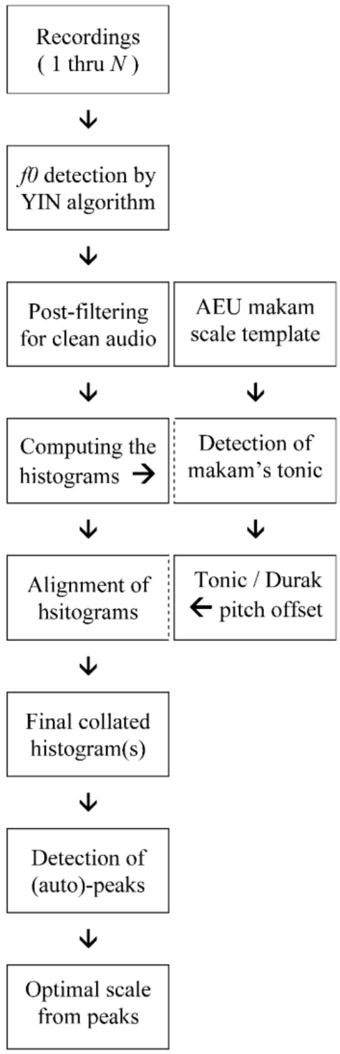
A summary of the frequency analysis procedure pertaining to both the older [Bozkurt et al., 2009] paper and the current contribution is presented in FHT 1. The addition of a peak detection algorithm to the earlier set of signal processing tools developed by Bozkurt [2008] for traditional Turkish music has allowed the straightforward extraction of the "optimal musical scale" from any digitized audio input against which to compare a theory. The entire algorithm was coded under MatLab by Bozkurt and is named as "*The MakamToolBox*".

The sample set upon which Bozkurt's batch operations were executed is, to reiterate, the same for either the [Bozkurt et al., 2009] paper and this contribution. In other words, the *makam* categories were and still are *RAST* (16 collated samples), *NİHAVEND* (12 collated samples), *KÜRDİLİHİCAZKAR* (17 collated samples), *UŞŞAK* (11 collated samples), *HÜSEYNİ* (15 collated samples), *HİCAZ* (17 collated samples) *SABA* (11 collated samples), *SEGAH* (16 collated samples) and *HÜZZAM* (13 collated samples), constituting a total of 128 audio files that led to the production of 9 distinct histogram plots. The dataset in question comprises performances with highly characteristic microtonal inflexions by Tanburi Cemil (*tanbur*, *kemençe*, violoncello), Mesut Cemil (*tanbur*, violoncello), Ercüment Batanay (*tanbur*), Fahrettin Çimenli (*tanbur*), Udi Hrant (violin), Yorgo Bacanos (*ud*), Aka Gündüz Kutbay (*ney*), Kani Karaca (vocal), Bekir Sidki Sezgin (vocal), Necdet Yaşar (*tanbur*), İhsan Özgen (*kemençe*) and Niyazi Sayin (*ney*); thus spanning a historical period from 1910 to 2001.

In FHT 1, I illustrated the automatic procedure used by Bozkurt's *The MakamToolBox* when analyzing a group of digitized audio files.

After feeding a given monophonic audio file into the code, one first finds the fundamental frequency *f0* and the rest of the pitches using the popular YIN pitch detection algorithm. One then sees that the *f0* raw data must be re-processed to fix the "mistakes" of YIN, such as correcting for the doubling/halving of octaves in this algorithm's mishandling of the Turkish *makam* zeitgeist, as well as to remove unwanted noise. Said post-filter too had been developed by Bozkurt [2008].

After the post-filter is complete, one sees that the corresponding histogram gets generated and is then calibrated in alignment with the best match to an Arel-Ezgi-Uzdilek *makam* scale template in order that the tonic of the performance is established for later collation with alike histograms.



FHT 1 Flowchart of the signal processing and histogram generation procedure that Bozkurt's *The MakamToolBox* operates on.

Here, the pitch histogram of concern is just a 2D drawing where the data table of detected frequencies gets dumped onto a graph representation based on their related incidences. Put in other terms, the histogram is basically a "90-degree hour-glassed melogram". Thus, the one-to-one correlation for the final plot is that where the *f0* values are categorized under bins according to the formula

$$Hf_0[n] = \sum_{k=1}^{K} m_k \ ,$$
$$with\ m_k = 1, f_n \leq f_0[k] < f_{n+1}$$
$$or\ m_k = 0, otherwise;$$

where the minima and maxima (f_n, f_{n+1}) represent boundary values that define the *f0* range for the n^th^ bin of our histogram representation.

The choice of the bin-width (f_{n+1}-f_n) – that is to say, the width of each category – demarcates the resolution of the histogram. For our purposes, it was, and for this contribution still is, desirable to use uniform sampling of the whole *f0* range. Through trial and error, the proper bin-width was established to be 1 degree of 159-tET (1/3^rd^ the Holderian comma, or *HC*, equaling 7.55 c), which is thought to optimize between the appearance of spurious peaks due to ultrafine discreteness and the undesirable loss of essential peaks on account of choosing a coarser grid.

One caveat must be mentioned at this point: All histogram graphics in the previous study[53] were originally drawn at 53 x 12 = 636-tET resolution as the lowest best-fit workable grid for the adequate representation of the whole of the evaluated theoretical models. This constrained misrepresentation to a maximum of less than 1 cent absolute error concerning any pitch of any theoretical model considered therein. Thus, all pitch histograms were 4-fold upsampled for merging. Since I am not redrawing histograms for my new analysis, it is not relevant here. It would be relevant if I or someone else decided to draw them based on the statistical findings hereunder.

Even so, the figures in Appendix C feature – for better or for worse – such 636-tET resolution plots from the previous study for visual comparison (albeit vertically squashed due to lack of publication space).

Another caveat is that, the MatLab code *The MakamToolBox* that was initially deployed by Bozkurt in our preparation of the original 2009 publication in the Journal of New Music Theory operated within the 0.33 Holderian comma (HC) sensitivity of the pitch measurement device. That is to say, for a scale tone at, say, 5 HC, all the frequency possibilities up to ±0.17 HC[54] around that value collapsed to 5 HC. All this goes without saying how the histogram auto-peaks would minutely diverge due to systematic machine errors if a separate pitch measurement run were to be conducted.

Nevertheless, the outcome would at any rate – and I stress this part – only diminutively differ from earlier pitch measurement runs, and would not affect the previous or current conclusions in any significate way.

After the histograms are aligned based on the *makam*'s tonic as per [Bozkurt, 2008], simple averaging of the histograms, where they are summed up and divided by their amount, as well as the normalization of the result according to the highest pitch incidence equaling "1", leads to the first of the two final histograms called the "average histogram". The second of the two final histograms uses, prior to normalization, the maximum function leading to what is called the "envelope histogram", that is otherwise the direct superposition of all histograms. A three-tap moving average filter then smoothens the ragged edges and allows for the robust detection of local maxima in terms of relative frequencies (*i.e.*, the so-called "autopeak-ave." and "autopeak-env."). These final histograms are presented together – albeit in a squashed way – on top of all the figures in Appendix C as I stated above for visual comparison against the findings of the current contribution.

Last, but not the least, mismatches between the two types of auto-peaks and the suggested scale tones of a theoretical model in any intervallic unit (such as *HC* or *cents*) can be shown. Knowing the relative frequencies I_{ai} of the autopeaks computed from the data automatically and the theorized scale tones I_{ti} of a tone-system, the maximum distance M and the average distance D between the two values for a given *makam* can be computed as

$$M = \max\{|I_{ai} - I_{ti}|\}, i = 1,2,3..N_x,$$
$$D = \frac{1}{N_x}\sum_{i=1}^{N_x}|I_{ai} - I_{ti}|,$$

where N_x is the total number of scale tones for a given *makam* out of all the pitches of the tuning that match a measured relative pitch. Everything else from this point onward regarding data evaluation is detailed under the Methodology and Code and Data sections of the present paper.

[53] [Bozkurt et al., 2009].

[54] Approx. 4 cents.

Appendix B

A new mandal *layout plan by Günalçin against the 79-tone tuning and Weiss's rational "Q9"* qanun

Serkan Günalçin recently spotlighted in his doctorate dissertation[55] a novel Just Intonation (JI) approach to installing *mandals* on the Turkish *Qanun* according to pitch measurements he meticulously gathered from 72 solo performances of Tanburi Cemil Bey. In the same spirit, Günalçin had previously defended[56] the abandonment of any equal tempered solution for this instrument after making a comparison with the electroacoustic assessments by Karl Signell[57] of idiosyncratic intervals between the frets of the late Necdet Yaşar's *Tanbur*. Yet, Günalçin too – like many others – incessantly keeps portraying my 79-tone strategy as a structure with entirely equal intervals[58]. I now wish to critique Günalçin's project in the light of the superior points of my 79-tone tuning and Stefan Pohlit's earlier doctoral work on the late Julien Jalaladdin Weiss's rational *Qanun* christened "Q9"[59]. (See for example the picture of my *qanun* in FHT 2.)

FHT 2 The author's *qanun* (bird's eye view).

In his exceptionally well-articulated doctorate study, Günalçin identifies the following crucial elements:

1. The *Qanun* has remained, even up-to-this-day according to some, a *nâ-kâmil* (under-developed) instrument that is incapable of correctly producing the genuine *perdes* of Turkish *Makam* music – unless when in the hands of legendary performers like Kanuni Hacı Arif Bey, who sucessfully determined accidented pitches with the pressure of their thumb nail or finger pinches on the string courses; (See an example of a *qanun* without *mandals* in FHT 3.)

FHT 3 "Kanun, and mode of playing it".[60] No *mandals* are to be seen on this instrument the way it had been depicted in mid-19th century.

2. Following the introduction of the advantageous *mandal* mechanism by the late 19th Century[61], the number of *mandals* per course increased dramatically throughout the decades (see for example the *mandals* on my *qanun* in FHT 4:87) – but even so, some renowned musicians shied away from the *Qanun* for its dearth of desired intonation at the initial stage;
3. Modern *Qanuns* today are mostly prepared in accordance with a voluminous and haphazard subset of sixfold stacked twelve-tone equal temperament resolution (for which Günalçin unfortunately still does not accord to me due credit in being the first person who drew attention, and with necessary technical detail, to the fact that *Qanun*-makers affix the semitone *mandal* at 100 cents owing to the prevalent usage of Eurogenuously manufactured electronic tuners, only to visually apportion the remaining distance

[55] [Günalçin, 2019]. (Also *cf.* TÜMAÇ BSE No. 12: "*Perde Anlayışı ve Kanun Mandallama Modelleri*", live-streamed webinar on 4 July 2020).

[56] [Günalçin, 2013].

[57] [Signell, 1977, p. 37–47, 151–161]

[58] [Günalçin, 2019, p. 11–12].

[59] [Pohlit, 2011 ; 2012].

[60] Retrieved from [Anon. "Datei:Kanun, and mode of playing it"], which refers to the original as [Thomson, 1859, p. 577].

[61] On the regular diatonically tuned *Qanun*, *mandal* technology was first implemented, according to Turkish musicologist Rauf Yekta, some 30 years prior to his submission of his invited monograph on Turkish Music to the 1922 edition of Albert Lavignac's *Encyclopédie de la Musique et Dictionnaire du Conservatoire* (*cf.* [Yekta, 1986, p. 92–93], and also *cf.* [Yarman et al., 2019]).

to the nut to basically 6 equidistant portions to arrive at virtually 72-tET[62]; [63]

FHT 4 The *mandals* on the author's *qanun*, as installed by the late Ejder Güleç, in accordance with the 79-tone system.

4. Because 72-tET does not approximate 53-tET degrees well – which, in turn, exceptionally houses Yekta-Arel-Ezgi-Uzdilek with only under 1 cent maximum absolute error at any degree[64] – and discrepancies as much as half a comma thereby arise, alternative approaches surfaced some decades ago, such as the 53-tET *mandal* scheme by Ethem Ruhi Üngör and the hybrid Pythagorean + equipartitioned *mandal* scheme by İsmail Baha Sürelsan, although they have mostly fallen out of notice in general when confronted with modern 12-tET nucleated *mandal* installation suggestions, such as by Sait Durgun (who defends 60-tET that I presently analyze in this study) and by Nail Yavuzoğlu (who previously defended 60-tET[65], but now defends 48-tET[66] – yet without giving any due mention to his predecessor Edward J. Hines – whereby 48-tET had already been debunked in [Bozkurt et al., 2009]);
5. Tanburi Cemil Bey is judged to be a ubiquitously-agreed-upon reference to pinpoint genuine Turkish *Makam* music intonation (although he is taken as the sole exemplar by Günalçin in contrast to the 12 distinct masters taken into consideration under the present study);
6. Praxis seems to involve the blend of two well-defined structures: (i) A Pythagorean plane for transpositions and modulations that must eventually be limited in scope because exact 3-limit intervals never conjoin at the octave, (ii) a greater prime-limit Just Intonation plane for capturing the *mujannabāt* [67] alongside special tetrachordal *genera* over certain degrees (regarding which the reader might wish to refer to [Yarman and Karaosmanoğlu, 2014] that I analyze herein as a clever attempt to bridge the commatic world with the quarter-tonal);
7. Rauf Yekta and Suphi Ezgi acknowledge the existence of these *mujannabāt*, whereas Hüseyin Saadettin Arel completely ignores them later on (which is somewhat of a false statement by Günalçin, because both Yekta and Ezgi specifically indicate that the "unsystematized" *mujannab* ratios of old are quite adequately superseded by the 24-tone Pythagorean tuning they espouse[68];

[62] This is to the extent that Günalçin's rather perfunctory statement in his webinar presentation (see fn. 55:86) that the microtones other than the *temam perdeha* (unaccidented notes or the naturals) are indeterminate is baseless in the face of all my elucidations since more than a whole decade.

[63] *cf.* ([Yarman, 2006; 2016; Yarman et al., 2019]. (Also *cf.* http://www.ozanyarman.com/79toneqanun.html.)

[64] [Yarman, 2010b, p. 22, 128, 130-31, 159].

[65] [Yavuzoğlu, 1991].

[66] [Yavuzoğlu, 2008].

[67] "*Mujannabāt*" (Pl. of *Mujannab* in Arabic): "The ones who are situated next to…", as had been first coined in extant Islamic music theory literature by al-Fārābī in his *Great Book of Music* (see [Beyhom, 2010, p. 205]), and later on picked up by Yalçın Tura as *mücenneb bölgesi* or the *mujannab zone* [Tura 1981], are a group of "semitonal to sesquisemitonal" fingerboard positions (being at least four in total, with all of them christened "*mujannab-i sabbaba*") bridgeward the open string on the *Oud* (while Amine Beyhom holds that frets or ligatures on the fingerboard of the medieval *Oud* might have solely been preferred for theoretical demonstrations and/or teaching purposes – *cf.* [Beyhom and Makhlouf, 2009]), corresponding to the pressure by the index finger at an anterior location compared to the standard whole tone location by the same finger (*i.e.*, *sabbaba*) (*cf.* [Sina, 2004, p. 109–112], [Kutluğ, 2000, p. 31–33], [Farmer, 1957, p. 456–464] and [Forster, 2010, p. 610–787]). They delineate a "microtonal continuum" in the pitch-space (with such a quotidian interpretation yet possibly amounting to merely an anachronism!), where especially the fractions 256/243 (90 c), 18/17 (99 c), 162/149 (145 c) and 54/49 (168 c) are associated with the feel of a "*mujannab* interval" (and where the latter two ratios, along with the addition of the Farabian 12/11 (151 c), are nearer to the contemporary sesquisemitonal understanding of this interval type).

[68] [Yekta, 2008, p. 13], [Ezgi, 1933, v. I, p. 57–57, 139; 1940, v. IV, p. 210–211]. (Also *cf.* https://islamansiklopedisi.org.tr/rauf-yekta-bey.) The reader is encouraged to consult the concomitantly cited references, where Yekta – for his prioritization of *Tahkiki* (well-determined – *i.e.*, *Pythagorean*) ratios instead of *Takribi* (approximate) ratios – plainly dismisses to notate 12/11 (his so-called "*nakıs büyük mücenneb*", or diminished large middle second), while Ezgi openly rejects even the possibility of 11/10 (let alone 12/11) in *Uşşak* and *Saba*. Needless to say, I was probably the first person to academically highlight the (politically calculated?) disregard of

8. Owing to the confluence of these facts, and despite the *Makam* music pitch-space being rather "unquantized" in practice, it is necessary to abandon 53-tET and combine a cornucopia of just intervals with Pythagorean ratios for especially the *Qanun*;

9. The intermarriage of Pythagoreanism with just intervals would restore the time-honored hint & clue relationship between praxis and theory, even with regards to previously available transpositions (such as Kâni Karaca's famous *Saba* modulation over *perde saba* while executing an ordinary *Saba*, as mentioned by Günalçin) whose historical links had been severed due to the draconian hemiolic limitations imposed by Arel (although the responsibility for such a rupture is actually shared by Yekta and Ezgi too the way I delineated in my doctorate thesis under the heading "*Rise of the 'Yekta-Arel-Ezgi School'*")[69];

10. Such an intermarriage seems already implied in the tractates of Urmavi[70], where Urmavi casually maps his JI[71] or highly complex rational *genera* to extended transpositions over his archetypal 17-tone Pythagorean grid – to the extent that one would then appear to find the need to shift frets on demand[72] (as happens to be just the subject I frequently communicated to colleagues like Prof. Arslan since more than a decade[73]), thus cementing the notion that the traditional Turkish music tone-system is open-ended instead of close-ended (although this is pretty much obvious so long as one is deprived of taking means geometrically or logarithmically!)[74];

11. Since transpositions are necessarily(!) Pythagorean[75] and plausible just intervals so numerous at any natural[76] degree, there is no way to avoid the emergence of an overwhelming mass of extra pitches in this open-ended system – whereby one must increase the *mandal* space while condoning some sacrifices in pitch exactness to gain the ability to transpose *makams* over the common *Ney Ahenks*[77];

12. Thusly, with occasional *mandal* revisions as much as 4.1 cents on the original "Model 1" proposal, one gains the ability to exceed the commonplace *Bolahenk Nısfiye*, *Kız* and *Mansur Ahenk* restrictions to achieve concert pitch additions such as *Müstahsen*, *Sipürde* and *Davud* (but not *Şah*?) – whereby, with the inclusion of double-sharp *mandals* (going all the way up to 23 *mandals* per course(!) compared to the 79-tone Qanun's maximum of 19), one then has 10+9(+5) *mandals* for *G*'s, 10+9 *mandals* for *A*'s, 10+6 *mandals* for *B*'s, identical 10+9(+5) *mandals* for *C*'s and *D*'s, selfsame 10+9 *mandals* for *E*'s, and homologous 10+6 *mandals* for *F#*'s (while, surprisingly enough, Günalçin does not exploit the opportunity to take here an *F* as the "natural" the way I had done) as space permits throughout A_2-E_5, resulting in a total of at least 91 distinct pitches per octave! – *i.e.*, 17 Pythagorean plus 74 just ratios[78];

such "acknowledged" out-of-the-ordinary macrotones by Yekta and Ezgi in my doctoral defense dated June 2008 (*cf.* [Yarman, 2016, p. 421]).

69 [Yarman, 2016, p. 15–24].

70 *cf.* [Arslan, 2007b ; 2007a].

71 "Just Intonation".

72 On instruments such as the *Tanbur*.

73 *cf.* [Yarman, 2007b, p. 3–4], redacted April 2011.

74 It is indeed regrettable that, although Günalçin on the one hand commends Yalçın Tura as having tackled the matter "in the best way" – *e.g.*, in terms of Tura's interpretation of Urmavi's 17-tone Pythagorean division as 17-tET at a pre-logarithmic era no less! – he keeps going on about the necessity for an open-ended tone-system for Classical Turkish music. While such a route might have been justified in the face of the Early Republican regime's targeting of "quarter-tones" as "Byzantine" and "Arabic" with an agenda to promote solely Western equal-temperament norms under the slogan of "Universal Music", why should there anymore be a *sine qua non* to forego Modern Age mathematical tools for the decent fractional exponentiation of a consonant interval to make things more regular and widely transposable? Günalçin's argumentation is clearly an outmoded ideological device under the post-logarithmic present day circumstances (especially after Michael Stifel's "*Arithmetica Integra*" [1544]).

75 Because of his Pythagorean Major scale on the "natural" *mandals* tuned by pure fifths, alongside an AEU-like extension over 3-limit sharps and flats.

76 *i.e.*, unaccidented.

77 "Concert Pitches" like *Bolahenk* with *perde rast* (second partial blown from all fingerholes of the Ney closed) at *D*; *Davud* with *rast* at *E*; *Şah* with *rast* at *F*; *Mansur* with *rast* at *G*; *Kız* with *rast* at *A*; *Müstahsen* with *rast* at *B*; and *Sipürde* with *rast* at *C*. Observe, that *perde rast* can be made to correspond to any tone of Western common-practice music, including all the half-tones in-between the naturals.

78 [Günalçin, 2019, p. 238–251].

13. "Enharmonic respellings" may then allow for transpositions at *Bolahenk-Sipürde mabeyni* (perde *rast* at $C^{\#}/D^{b}$) and *Şah-Mansur mabeyni* (perde *rast* at $F^{\#}/G^{b}$), as well as other far-off keys (which Günalçin baselessly claims is a novelty not thought of or considered before – although my 79-tone *Qanun* tuning recipe [79] based on enharmonic equivalences that he himself outlined[80] is a stark refutation of such a claim);

14. Given the "purity and authenticity of intonation" Günalçin purportedly achieves through his ponderous method, which he personally implemented on a *Qanun* whose mainbody was manufactured by Mustafa Sağlam (that involves some incomprehensible latent *mandal* additions and revisions under his "Model 2" [81]), other implementations such as my 79-tone *Qanun* do not, according to him, reflect practice just as well (despite the fact that Günalçin verbally[82] admits 79 MOS 159-tET as very closely verging on *all* of his Pythagorean and "natural" ratios)[83];

In a broad sweep, the weaknesses of Günalçin's point of departure are as follows…

He presumes his solitary exemplar (Tanburi Cemil Bey) is authoritatively representative of all of Turkish Classical/Art/Folk music – such as regarding the supposed absence, in praxis, of the 256/243 *leimma* amounting to an interval size of 90 c (except when one goes up some fifths and plays at a different key, or for completing a tetrachord) that he interprets is Cemil Bey's escape attempt from Pythagoreanism. Yet, were he to include other masters who, in particular, perform on different instruments – as is done under the present study – Günalçin would probably have witnessed the happenstance of such a *leimma* regularly, as well as the execution of tetrachords and pentachords in his list of "proper genera" over many more tones than he gives license[84].

He dismisses the merits of other theoretical models without investigating their success in matching the pitch measurements from his chosen exemplar. In other words, he does not weigh the body of historical *mujannabāt* ratios[85] and tetrachordal divisions that agree with his pitch measurements against the thus-far implemented *Qanun mandal* configurations of Üngör, Sürelsan, Durgun, Weiss and Yarman, to say nothing of the "*piyasa Kanun'u*" (12-tET nucleated "marketplace *Qanun*"). Out of all of these, the intonational success of especially 72-tET as well as the Weiss and Yarman implementations are not at all properly scrutinized – even though Günalçin particularly states that the 79-tone tuning very closely approximates the "natural" intervals that are under his focus[86].

It would seem here that Günalçin wants to give the impression that Yekta and Ezgi actually condoned the usage of *mujannabāt* outside the bounds of their 24-tone Pythagorean cast – and hence, extant tetrachordal *genera* featuring such "middle seconds" – while Arel supposedly unilaterally forbade them altogether. Not only is this an inadmissible anachronism, but it is plainly wrong; and so is Günalçin's assertion that Yekta actually expressed the applicability of Greek-Islamic *genera* to quotidian praxis instead of seeing them confined to dusty books and shelves[87].

[79] [Yarman, 2016] & [Yarman, 2010a, p. 60]. (Also *cf*. [Moriarty, 2014].)

[80] [Günalçin, 2019, p. 12].

[81] [Günalçin, 2019, p. 247–251].

[82] In his webinar cited under footnote 55:86.

[83] Regrettably once more, Günalçin asserts that one is obliged to explain the tone-system of Classical Turkish music by either dividing the octave "equally" (why?!), or by limiting the number of ratios, or by sanctioning an excess of Pythagorean and just *mandals* for extended transpositions. Yet, as it so happens, 79 MOS 159-tET is not an entirely equal construct (*e.g.*, it partitions 4/3 instead into 33 equal portions) while still permitting regularized and tolerable transpositions over *all* degrees. This only goes to show that the prevailing mindset is inexcusably "pre-logarithmic" or "12-tET nucleated", seeing as Turkish *Qanuns* are, for the most part, still being prepared in accordance with multiples of twelve-tone equal temperament the way I had first described, or "traditionalist academical alternatives" are, almost without exception, inclined toward an arithmetic historism of fractional interval usage or an equipartitioning of the 100 c "equal semi-tone" as referred to Eurogenuous electronic tuners or popular score engraver computer programs.

[84] *cf*. [Günalçin, 2019, p. 176, 182, 193, 215].

[85] *cf*. [Arslan, 2007b ; 2007a, p. 336]

[86] *cf*. the webinar link in footnote 55:86.

[87] On page 63 of his aforementioned monograph (see notably [Yekta, 1986, p. 60–63]), Yekta contrariwise writes (English translation and emphases are mine): "It may thence be asked of us 'How many harmonious tetrachords have been obtained (by Hellenistic and Islamic music theorists) after such tiresome labors?' To this we reply, and with some embarrassment, ONLY FOUR (*i.e.*, A. **9/8 x 9/8 x 256/243**, B. **9/8 x 10/9 x 16/15**, C. **15/14 x 7/6 x 16/15**, D. **12/11 x 7/6 x 22/21**) that have been judged as consonant. The others have been relegated to remain in tractates; yet, even so, after so many centuries, they have muddled the minds of European theorists wanting to apply them in unproductive works whilst desiring to demystify the enigma of *genera* in Greek music!".

Günalçin similarly misrepresents Tanburi Cemil Bey when he quotes him as allegedly saying "*Tanbur* is a fretless instrument"; whereas Cemil Bey rather states (English translation is mine):

> "… These frets are not fixed upon the neck as is the case with Mandolin and Guitar, and are oiled in such a way as to be able to be moved either way while there is enough space on the neck to allow for this and that any desired fret may be added to the *Tanbur*. The *Tanbur* thence possesses the means of expression unique to fretless instruments as such."[88]

In other words, contrary to what Günalçin asserts, the re-positioning and/or increase of frets serves to "microtonalize" the *Tanbur*, but not really to make it "fretless per se"; as is also the case with the *Qanun* via minutiae granted by a multiplicity of fixed *mandals*. So, Günalçin does not truly answer the question: "why exceed Cemil Bey's more-or-less-predetermined 36 *Tanbur* frets per two octaves[89] if one is ultimately limiting oneself in terms of a few transpositions and modulations over a few *Ahenks*?"

Günalçin hence assumes transpositions and modulations must eventually be limited on the pretext that an infinite number (*sic*) of "natural" ratios can be advanced and that the traditional Turkish music tone-system should be open-ended (*i.e.*, not be "cyclic", and not therefore be based on tempered perfect fifths). Yet, he does not seem to realize that such a presupposition rests solely on the premise of totally neglecting tolerable geometric and logarithmic divisions when equipartitioning simple epimoric consonant intervals like 2/1 or 3/2. Given that the danger of getting the tradition branded as "Arabic" or "Byzantine" with an intent to cast it out of society no longer exists, Günalçin has no visible recluse to freeze the transpositional and modulational development of *Makam* music at late 19th and early 20th Centuries so as to extol an itinerary of pure arithmeticism in music theory compared to higher mathematical devices as in taking roots or taking logarithms, which are well considered to be beneficial worldwide in music-making and toward the advancement of music in general.[90]

He thus opines, and without any basis whatsoever, the precedence of a pre-logarithmic mindset to a post-logarithmic mindset in *Makam* music; imagining moreover that post-logarithmic options solely constitute "equal divisions of the octave". He, just like many of my peers, does not seem to understand the structure and function of especially the "hybrid modified meantone cores" of my 24-tone and 36-tone tuning suggestions.[91]

Furthermore, Günalçin does not appear to genuinely exploit "enharmonic equivalences" when he affixes as much as 23 *mandals* per course. It only suffices to visually compare his scheme (he unfortunately does not yet provide a complete list of his ratios and/or temperings in an octave) with the nominal 19 *mandals* per course on the 79-tone *Qanun* (with my last 4 *mandals* serving as double-sharps). Surely, any criticism as to "unplayability" in the case of the 79-tone *Qanun* applies with even greater force against Günalçin's "Model 1" or "Model 2" given his minimum of 91 pitches per octave!

When all is said and done, Günalçin's smallest occurring interval between any two neighboring *mandals* is an impracticably tight-packed 7.71 c[92]. For people criticizing how the 79-tone *Qanun*'s smallest interval (15.1 c) in comparison to 72-tET's smallest (16.7 c) is troubling from the viewpoint of musical performance, half of what I propose at already the very limits of *mandal* installation for the register A_2-E_5 is without any doubt much more unfeasible.

Despite so much detail, Günalçin's configuration – due to the curtailed Pythagorean tuning at its core – cannot let the sharps and flats meet and overlap circularly (*e.g.*, through a cycle-of-fifths) should the need arise (such as in cases of chromaticism or in accompaniment with Western-style ensembles and orchestras). In contradistinction, the 79-tone tuning permits this[93]: For example, mode 6 7 7 6 7 6 7 6 7 7 6 7 of 79 MOS 159-tET, equaling 91 + 106 + 106 + 91 + 106 + 91 + 113 + 91 + 106 + 106 + 91 + 106 consecutive cents, extracts a workable 12-tone cyclic subset – which is one of several alike possibilities – and contains only one "wolf fifth" that may very well be considered tame. (In addition, it

[88] [Cemil Bey, 1993, p. 22].

[89] [Günalçin, 2019, p. 134-137].

[90] (Note from the Editors:) NEMO-Online does not concur with such a statement, but accepts the author's point of view on this matter.

[91] And even though all my labors are accessible through my personal website (http://www.ozanyarman.com) and YouTube channel (https://www.youtube.com/user/DrOzanYarman) for at least a decade.

[92] [Günalçin, 2019, p. 243-246].

[93] [Yarman, 2016, p. 115–116].

is conceivable to satisfactorily encapsulate Arel-Ezgi-Uzdilek as mode 6 1 5 1 7 1 4 2 4 2 6 1 5 1 6 1 5 1 7 1 4 2 4 2 under 79 MOS 159-tET, which thus equals 1+15+76+15+106+15+61+30+61+30+91+15+76+21+91+15+76+15+106+15+61+30+61+31 consecutive cents.)[94]

And while 79 MOS 159-tET can approximate every one of Günalçin's list of *Djemilian* tetrachords (*i.e.*, derived from Tanburi Cemil Bey) in the Tables 4.80 & 4.81 of his doctorate dissertation[95] with a maximum of 7-8 cents absolute error at any given degree[96], the added bonus of unlimited transpositions cannot be matched with his "Model 2" – let alone his "Model 1".

Finally, Günalçin acquiesces to adopt the inconsistent Arel-Ezgi-Uzdilek notation (*i.e.*, the Turkish Classical/Art music accidentals symbolism in effect) for his irregular megalithic edifice[97]. However, by doing so, he pulverizes any venue for executing advanced or experimental techniques that might involve calculated xenharmony[98] on his *Qanun*. In such a way, Günalçin prevents his *Qanun* from becoming an instrument that may serve to expand and cosmopolitanize microtonal musical expression while preserving tradition – especially in the face of institutional and constitutional modern education requirements. In contrast, the 79-tone *Qanun* is accompanied by a re-purposed complementary "Sagittal Notation" originally developed by David Keenan and the late George Secor, which can hence serve as a gateway to future "*makam* polyphony" while being backwards-compatible with 65-tET and 72-tET[99].

I now propose to compare Günalçin's *rast-çargâh* tetrachordal layout with Weiss' commensurate "Q9" tetrachordal layout. (See THT 1:93.)

Shaded cells are either out of range or unavailable. For instance, the 225 c, 242 c and 257 c *mandals* are omitted on both "Q8" and "Q9" to expedite "flexibility and ease of modulation"[100], but they nevertheless remain relevant theoretically and are described at any rate. Likewise, there are no correspondences in Günalçin (according to his nominal *Bolahenk* starting on D_4) for the 46/45, 32/32 and 91/87 *mandals* found on "Q9" (according to the nominal *Sipürde Ahenk* starting on C_4), and I disregarded to evaluate them for the sake of fairness.

Julien Weiss' "Q9", just like his previous "Q8" implementation (with inconsequential differences in regards to some convergent intra-*mandal* ratios), contains a standard of 14 *mandals* per course throughout (except the aforesaid omissions under some string courses), and Stephen Pohlit[101] gives the particulars of this rational layout in one octave as both fractions and cents. What is relevant here for our purposes is the tetrachordal span starting on *perde rast* (C_4).

By comparison, I was able to extract the tuning information out of what Günalçin dubs "*mandal sets 1-3*"[102] through considerable difficulty and via checking and re-checking his online presentation. These correspond to *Fa/do* (C_4/G_4) for set 1, *Sol/ré* (D_4/A_4) for set 2, and *La/Si/mi* ($E_4/F^{\#}_4/B_4$) for set 3. Extended revisions (constituting his "Model 2") lead to $E_4/F^{\#}_4/B_4$ being assigned to the "modified set 1" (now having 23 *mandals* instead of the previous 18!), C_4/G_4 being assigned to the "modified set 2" (now having 19 *mandals* instead of the previous 15 or 16), and D_4/A_4 being assigned to the "modified set 3 (now having 16 *mandals* instead of the previous 15).

Closest matches of "Q9" ratios (in bold) have thus been juxtaposed against Günalçin's ratios that I extracted unto THT 1:93. Notice that the largest absolute difference (marked with red) in-between them for any *perde* occurrence is less than 8 cents (while any kind of average of the modulus of differences is about 3 to 6 cents) – just as in the case of the approximation capabilities of my 79-tone *Qanun* tuning. Such is, to all intents and purposes, near or at the "acoustic limen"[103] for the entirety of the pragmatic ambitus of the *Qanun*.

94 *cf.* https://www.youtube.com/watch?v=gaE2x-dQtBQ (Dr. Oz. @ Anadolu Üniversitesi Math. Dept. - 7 November 2014).

95 [Günalçin, 2019, p. 193, 215].

96 [Yarman, 2016, p. 95-100].

97 [Günalçin, 2019, p. 239, 257].

98 The term "*xenharmonic music*", the way originally coined by Ivor Darreg [see https://en.wikipedia.org/wiki/Xenharmonic_music], entails provocative non-12 (*detwelvulated*) harmonies such as found under 17-tET or 19-tET, or perhaps more appropriately yet under 5-tET, 7-tET, 11-tET, 13-tET, etc... It is almost the same as saying "*microtonal polyphony*", except that one leaves open the door for more courageous "dissonant-consonant adventures"; which happens to connote a field ripe for exploration when one is faced with so many rich tuning choices under *Makam* / *Maqam* / *Mugham* / *Muqam* music by and large.

99 [Yarman, 2016, p. 106-109].

100 [Pohlit, 2011, p. 76–77].

101 [2011, p. 262–264].

102 [Günalçin, 2019, p. 243-251] & webinar link.

103 [Long, 2014, p. 81–127]. pp. 81-127: The acoustic limen is just about 7 cents on average throughout the common musical range

This basically signifies that the whole enterprise by Günalçin – despite his painstaking efforts to uphold *makam* fidelity – practically amounts to what Weiss had already accomplished with even lesser *mandals* (at yet the expense of the lack of double-sharps as well as the 28/27 approach to the natural of the lower course when all *mandals* of the upper course are lowered) – aside from the fact that either choice can be much more advantageously substituted with 79 MOS 159-tET. This is especially true for the latter's ability to approximate – and I cannot stress this part enough – any given ratio from either former case at any level of transposition with only a maximum of 7-8 cents absolute error; not to mention the 79-tone tuning's avoidance of reckless schismatic aberrations – such as the 500 cent 10935/8192 ratio in both former cases not being exactly enharmonically equivalent to the 498 cent 4/3 ratio at their upper courses.

Mind that Günalçin moreover appears to have altered his doctoral scheme for the upended "*mandal* set 2" corresponding to the G&C courses[104] in order to squeeze in one more *mandal* at only 5.76 c away from its nearest neighbor! I highlighted its location in yellow in THT 1:93.

To wrap up this sub-section, I wish to point out the instructional video titled "*Tuning Theory 3: Moment of Symmetry ("Microtonal" Theory) --- John Moriarty*" by John Moriarty from the "Xenharmonic Alliance" in order to better illuminate the concept of "Moment-of-Symmetry" ("MOS", as had been first coined and articulated by Ervin Wilson), since it happens to be an intrinsic and essential feature of the 79-tone tuning[105]; which owes its mathematical integrity and transpositional regularity as such to the presence of just two (one large "L" and one small "S") types of commas derived from a single generator interval (*e.g.*, seventy-eight instances of 15.1 c and a single final instance of about 22 c occurring throughout *yegah-neva* coincident to *Sipürde Ahenk* at G_3-G_4).

For a recapitulation of the unique aspects of the 79-tone *Qanun* tuning, I outline in THT 2:94 the octave structure and reasonably adequate fractional approximation possibilities under 79 MOS 159-tET:

The 79-tone *Qanun* tuning thus distinctively embodies the following intertwined assets:

1. A theoretically proper *Rast* "ascending scale" can be mapped to a "Just Intonation C Major" without breaking the chain of fifths via using two different kinds of perfect fifths (694 c and 702 c) in the manner *F* (702 c) *C* (702 c) *G* (694 c) *D* (702 c) *A* (694 c) *E* (702 c) *B*;
2. Such a *Rast* scale can be transcribed on the staff (even when using the complementary Sagittal Notation) without any accidentals to begin with in compliance to the historical notion of *Rast* being the "mother-of-all-*makams*"[106] – delineating therefore the primacy of *Sipürde Ahenk* as the gravitational center of all transpositions (instead of *Bolahenk*) where *perde hüseyni* corresponds to A4 = 440 Hz;
3. Moreover, a theoretically proper *Mahur* "ascending scale" can similarly be mapped to a "Pythagorean *C* Major" instead without again breaking the generator chain of fifths – via relying on three different kinds of perfect fifths this time (694 c, 702 c and 709 c) in the manner *F* (702 c) *C* (702 c) *G* (694 c) *D* (702 c) *A* (709 c) *E*‡ (702 c) *B*‡, where such an unbroken modulation follows from the direct alteration of *segâh-eviç* to *buselik-mahur* (I hesitate regarding the "*Arelian*" *mahur*, think it to be 1 degree lower at *eviç*, and its fifth to be *segâhçe*);
4. Likewise, *dügâh-hüseyni* can be alterated up by one *mandal* each to facilitate yet another Major scale without breaking the chain of fifths;
5. Even once more, the *buselik-mahur* perdes can be altered up by yet another *mandal* each to facilitate a "Super-Pythagorean *C* Major";
6. This described feature can be repeated over several keys, where – in agreement with traditional guidelines – *buselik* and *mahur* become accidented pitches (*nim perdeler*) and are notated as such;
7. One can also turn to a few distinct options to extract a cyclic subset for chromaticism – where, in particular, mode 6 7 7 6 7 6 7 6 7 7 6 7 out of 79 MOS 159-tET, equalling 91 + 106 + 106 + 91 +

for a moderately audible volume as given in Table 3.4 of the cited source. {Here, the issue is not the ability of the dilettante to discern if something is awry with a singular interval in suspense; the issue is rather his/her propensity to tolerate tuning errors (*i.e.*, "temperings") in musical flow. Also *cf.* https://www.sciencedirect.com/topics/engineering/just-noticeable-difference.}

[104] [Günalçin, 2019, p.] & webinar link.

[105] *cf.* [Yarman, 2010a, p. 32–63] and www.ozanyarman.com/files/searchfortheoptimaltonesystem.zip. (Also *cf.* [Moriarty, 2014] on "MOS".)

[106] *cf.* [Levendoğlu, 2003].

106 + 91 + 113 + 91 + 106 + 106 + 91 + 106 consecutive cents, extracts a workable 12-tone cyclic subset (*e.g.*, with overlapping "sharps" and "flats") which contains only one "wolf fifth" that can very well be considered tame;

8. In addition, it is conceivable to adequately encapsulate Arel-Ezgi-Uzdilek too as mode 6 1 5 1 7 1 4 2 4 2 6 1 5 1 6 1 5 1 7 1 4 2 4 2 under 79 MOS 159-tET, which thus equals 91 + 15 + 76 + 15 + 106 + 15 + 61 + 30 + 61 + 30 + 91 + 15 + 76 + 21 + 91 + 15 + 76 + 15 + 106 + 15 + 61 + 30 + 61 + 31 consecutive cents (where "sharps" and "flats" do not any more overlap);
9. Lastly, the overarching Sagittal Notation[107] consistently maps the quarter-tone to 3 steps (/|\, \|/), the *leimma* to 6 steps (#, b), the *apotome* to 7 steps (/|#, \|b), etc. – wherefore xenharmony and *makam* polyphony is encouraged, since all kinds of intervals (and especially the *mujannabāt*) can be situated at any level of transposition, be they inside or outside tetrachordal or pentachordal genera.

In the light of the foregoing theoretical and physical drawbacks for Günalçin, and especially with regards to such an inordinate *mandal* inflation gaining us fundamentally next to nothing, which moreover ought to render the *Qanun* quasi-functional and inoperable in practice, it is apparent to me that Günalçin's project falls short of its goals in comparison to the quotidian alternatives at hand. I henceforth ignore it in this study.

* * *

çargah	8192/6561	512/405	80/63	448/351	104/81	128/99	176/135	21/16	320/243	*4/3 G4*
CENTS	384	406	414	422	433	445	459	471	477	*498*
çargah	**8192/6561**	**512/405**		**448/351**	**2816/2187**	**4096/3159**	**952/729**	**320/243**		***4/3***
CENTS	**384**	**406**		**422**	**438**	**450**	**462**	**477**		***498***
DIFF.	0	0	8	0	-5	-5	-3	-6	0	0
buselik	7/6	32/27	6/5	135/112	63/52	39/32	27/22	99/80	5103/4096	*81/64 F#4*
CENTS	267	294	316	323	332	342	355	369	381	*408*
buselik	16…	**32/27**	**6/5**		**63/52**	**11/9**	**16/13**	**119/96**	**5/4**	***81/64***
CENTS	269	**294**	**316**		**332**	**347**	**359**	**372**	**386**	***408***
DIFF.	-2	0	0	7	0	-5	-4	-3	-5	0
dügah	28/27	256/243	16/15	15/14	14/13	13/12	12/11	11/10	567/512	*9/8 E4*
CENTS	63	90	112	119	128	139	151	165	177	*204*
dügah	16…	**256/243**	**16/15**		**14/13**	**88/81**	**128/117**	**119/108**	**10/9**	***9/8***
CENTS	65	**90**	**112**		**128**	**143**	**156**	**168**	**182**	***204***
DIFF.	2	0	0	7	0	-4	-5	-3	-5	0
rast	224/243	2048/2187	128/135	20/21	112/117	26/27	32/33	44/45	63/64	80/81
CENTS	-141	-114	-92	-84	-76	-65	-53	-39	-27	-22
rast									-5 c!	
CENTS										
DIFF.										

çargah	27/20	1215/896	567/416	351/256	243/176	891/648	45/32	729/512	81/56					
CENTS	520	527	536	546	557	573	590	612	639					
çargah														
CENTS														
DIFF.														
buselik	6561/5120	295245/229376	137781/106496	85293/65536	59049/45056	216513/163840	10935/8192	177147/131072	19683/14336	45927/32768	1476225/1048576	33014845/16777216	59049/40960	43046721/29360128
CENTS	429	437	446	456	468	483	500	522	549	584	592	614	633	662
buselik	6561/5120		137781/100496	26733/2048	**2187/1664**	**86751/65536**	**10935/8192**	177147/131072						
CENTS	429		446	461	**473**	**486**	**500**	522						
DIFF.	0	8	0	-5	-5	-3	0	0						
dügah	729/640	32805/28672	15309/13312	9477/8192	6561/5632	24057/20480	1215/1024	19683/16384	2187/1792	5103/4096	164025/131072	2657205/2097152	6561/5120	4782969/3670016
CENTS	225	233	242	252	264	279	296	318	345	381	388	410	429	459
dügah	729/640		15309/13312	297/256	**243/208**	**9639/8192**	**1215/1024**	**19683/16367**	upper	upper	upper	upper	upper	upper
CENTS	225		242	257	**269**	**282**	**296**	**319**	347	386	386	408	429	461
DIFF.	0	8	0	-5	-5	-3	0	-1	-2	-5	2	2	0	-2
rast	*D4*	64/63			28/27	25/24		135/128	16/15	243/224				
CENTS	*0*	27			63	71		92	112	141				
rast	***C4***	**81/80**	**46/45**	**33/32**	**27/26**		**91/87**	**135/128**	**2187/2048**	upper				
CENTS	***0***	**22**	**38**	**53**	**65**		**78**	**92**	**114**	143				
DIFF.	0	5			-2	6		0	-2	-2				

Maximum absolute diff.: **7.71 cents (c)**
Average of abs. diffs. ≈ 150/58 = **2.74 c**
Root-mean-square ≈√ (760/58) = **3.78 c**
Harmonic mean of abs. diffs. = **5.81 c**
Balanced sumtotal average ≈ **-42/58 = -0.89333 c**

THT 1 Comparison of the *rast-çargah mandal* area between Günalçin's "Model 2" and Weiss' commensurate "Q9" rational *Qanun* (in bold).

[107] (Note from the Editors): For readers not familiar with this notation, references [Keenan, 2004 ; Secor and Keenan, 2006 ; Xenharmonic Wiki Contributors, 2020] provide useful and comprehensive information.

Degree	Integer Cents	159-tET subset	17 "traditional" perde zones	Simplified Frequencies	Cents	Frequencies (A4=440Hz)	Cents	Consecutive intervals	Approximated ratios with at most 7.5 cents error
0:	**0**	**0**	***Rast - C***	262	**0.000**	261.911	**0.000**	*(with prev.)*	**1/1 (Do)**
1:	15	2	*dik rast*	264 1/2	16.441	264.204	15.094	15.094	126/125,100/99,81/80
2:	30	4	*(sarp rast)*	266 3/4	31.106	266.518	30.189	15.095	64/63,3125/3072,55/54
3:	45	6	*(rast+irha)*	269	45.647	268.852	45.283	15.094	128/125,36/35,33/32
4:	60	8	*(alt şûri)*	271 1/4	60.068	271.206	60.377	15.094	729/704,28/27,27/26
5:	75	10	*şûri*	273 3/4	75.951	273.581	75.472	15.095	25/24,117/112,22/21
6:	91	12	*nim zengûle*	276 1/4	91.689	275.977	90.566	15.094	20/19,256/243,135/128
7:	106	14		278 1/2	105.733	278.394	105.660	15.094	17/16,16/15,2187/2048
8:	121	16		281	121.204	280.832	120.755	15.095	15/14,14/13
9:	136	18	*zengûle* cluster	283 1/2	136.538	283.291	135.849	15.094	14/13,27/25,13/12
10:	151	20		286	151.738	285.771	150.943	15.094	88/81,12/11,35/32
11:	166	22		288 1/2	166.805	288.274	166.038	15.095	11/10,54/49
12:	181	24	*dik zengûle*	291	181.743	290.798	181.132	15.094	65536/59049,10/9
13:	**196**	**26**	***Dügâh - D***	293 2/3	**197.535**	293.345	**196.226**	15.094	28/25,**9/8 (Re)**
14:	211	28	*dik dügâh*	296	211.236	295.914	211.321	15.095	9/8,26/23
15:	226	30	*(sarp dügâh)*	298 3/4	227.246	298.505	226.415	15.094	256/225,8/7
16:	242	32	*nim kürdi*	301 1/2	243.109	301.119	241.509	15.094	144/125
17:	257	34	*(nim nihâvend)*	304	257.405	303.756	256.604	15.095	37/32,81/70,125/108
18:	272	36	*alt kürdi*	306 3/4	272.996	306.416	271.698	15.094	7/6
19:	287	38	*kürdi*	309 1/4	287.048	309.099	286.792	15.094	33/28,13/11,32/27
20:	302	40	*dik kürdi*	312	302.375	311.806	301.887	15.095	32/27,25/21,81/68
21:	317	42	*nihâvend*	314 3/4	317.567	314.536	316.981	15.094	6/5,19683/16384
22:	332	44	*hicâzî segâh*	317 1/2	332.628	317.291	332.075	15.094	63/52,40/33,17/14
23:	347	46	*uşşâkî segâh*	320 1/4	347.558	320.069	347.170	15.095	39/32,11/9,27/22
24:	362	48	*sâbâî segâh*	323	362.361	322.872	362.264	15.094	16/13,100/81,21/17
25:	377	50	*Segâhçe* (***Segâh*** *swap?)*	326	378.366	325.699	377.358	15.094	31/25,41/33,46/37,5/4
26:	**392**	**52**	***Segâh****(çe?)* ***- E***	328 3/4	**392.909**	328.552	**392.453**	15.095	**(Mi) 5/4**,64/51,59/47
27:	408	54	*bûselik (E‡)*	331 3/4	408.636	331.429	407.547	15.094	81/64,19/15,33/26
28:	423	56	*nişâbür*	334 1/2	422.927	334.331	422.642	15.095	14/11,23/18,32/25
29:	438	58	*(dik nişâbür)*	337 1/2	438.385	337.259	437.736	15.094	9/7
30:	453	60	*(bûselik+irha)*	340 1/2	453.706	340.212	452.830	15.094	35/27,13/10
31:	468	62	*(nişâbür+irha)*	343 1/2	468.892	343.191	467.925	15.095	38/29,21/16
32:	483	64	*alt çargâh*	346 1/2	483.946	346.197	483.019	15.094	33/25,37/28
33:	**498**	**66**	***Çargâh - F***	349 1/3	**498.045**	349.228	**498.113**	15.094	**4/3 (Fa)**
34:	513	68	*dik çargâh*	352 1/2	513.668	352.287	513.208	15.095	39/29,35/26,27/20
35:	528	70	*(sarp çargâh)*	355 3/4	529.556	355.371	528.302	15.094	19/14,49/36
36:	543	72	*nim (garip) hicaz*	358 3/4	544.094	358.483	543.396	15.094	26/19,48/35,11/8
37:	558	74	*(nim sâbâ)*	362	559.707	361.623	558.491	15.095	11/8,29/21
38:	574	76	*alt hicaz*	365	573.996	364.789	573.585	15.094	25/18,32/23,39/28
39:	589	78	*hicaz*	368 1/4	589.342	367.984	588.679	15.094	7/5,1024/729,45/32
40:	604	80	*uzzâl*	371 1/2	604.554	371.206	603.774	15.095	24/17,17/12
41:	619	82	*sâbâ*	374 3/4	619.634	374.457	618.868	15.094	10/7
42:	634	84		378	634.583	377.736	633.962	15.094	23/16,36/25,49/34
43:	649	86	*sâbâ-bestenigâr* cluster	381 1/4	649.405	381.044	649.057	15.095	16/11,8192/5625,35/24
44:	664	88		384 3/4	665.225	384.381	664.151	15.094	22/15,69/47,72/49
45:	679	90		388	679.788	387.746	679.245	15.094	37/25,40/27
46:	**702**	**93**	***Nevâ - G***	393	**701.955**	392.851	**701.887**	22.642	**3/2 (Sol)**
47:	717	95	*dik nevâ*	396 1/4	716.213	396.291	716.981	15.094	53/35,50/33,1024/675
48:	732	97	*(sarp nevâ)*	400	732.520	399.761	732.075	15.094	32/21,29/19,75/49
49:	747	99	*(nevâ+irha)*	403 1/2	747.602	403.262	747.170	15.095	192/125,20/13,54/35
50:	762	101	*(alt bayâti)*	407	762.554	406.793	762.264	15.094	45/29,59/38,14/9
51:	777	103	*bayâti*	410 1/2	777.378	410.356	777.358	15.094	25/16,47/30,11/7
52:	792	105	*nim hisar*	414	792.077	413.949	792.453	15.095	30/19,128/81,19/12
53:	808	107		417 1/2	806.651	417.574	807.547	15.094	43/27,8/5,6561/4096
54:	823	109		421 1/4	822.132	421.231	822.642	15.095	37/23
55:	838	111	*hisar/hüzzam* cluster	425	837.475	424.919	837.736	15.094	34/21,81/50,13/8
56:	853	113		428 3/4	852.684	428.640	852.830	15.094	44/27,18/11,105/64
57:	868	115		432 1/2	867.760	432.394	867.925	15.095	28/17,33/20
58:	883	117	*dik hisar* (*hisârek*)	436 1/4	882.706	436.180	883.019	15.094	32768/19683,5/3
59:	**898**	**119**	***Hüseyni - A4***	**440**	**897.524**	**440.000**	**898.113**	15.094	5/3,42/25,**27/16 (La)**
60:	913	121	*dik hüseyni*	444	913.191	443.853	913.208	15.095	27/16,39/23,17/10
61:	928	123	*(sarp hüseyni)*	447 3/4	927.752	447.740	928.302	15.094	128/75,41/24,12/7
62:	943	125	*nim acem*	452	944.107	451.661	943.396	15.094	50/29,216/125,64/37
63:	958	127	*(nim dik acem)*	455 3/4	958.411	455.616	958.491	15.095	125/72,40/23,47/27
64:	974	129	*alt acem*	459 3/4	973.539	459.606	973.585	15.094	7/4,225/128
65:	989	131	*acem*	463 3/4	988.537	463.630	988.679	15.094	23/13,16/9
66:	1004	133	*dik acem*	467 2/3	1003.097	467.691	1003.774	15.095	16/9,25/14
67:	1019	135	*sarp acem*	472	1019.064	471.786	1018.868	15.094	9/5,59049/32768
68:	1034	137		476	1033.674	475.917	1033.962	15.094	29/16,20/11
69:	1049	139	*eviç/arazbar* cluster	480 1/4	1049.063	480.085	1049.057	15.095	11/6
70:	1064	141		484 1/2	1064.316	484.289	1064.151	15.094	37/20,50/27,13/7
71:	1079	143	*alt eviç* (***Eviç*** *instead?)*	488 1/2	1078.550	488.530	1079.245	15.094	28/15
72:	**1094**	**145**	***Eviç*** *(Mâhur swap?)* ***- B***	492 3/4	**1093.547**	492.808	**1094.340**	15.095	**(Si) 15/8**,32/17,17/9
73:	1109	147	*(dik?) mâhur (B‡)*	497 1/4	1109.285	497.124	1109.434	15.094	256/135,243/128,40/21
74:	1125	149	*dik(çe?) mâhur*	501 1/2	1124.019	501.477	1124.528	15.094	21/11,23/12,48/25
75:	1140	151	*(mâhurek)*	506	1139.485	505.868	1139.623	15.095	27/14,29/15,31/16
76:	1155	153	*(dik mâhurek)*	510 1/4	1153.965	510.298	1154.717	15.094	37/19,39/20,125/64
77:	1170	155	*(sarp mâhurek)*	514 3/4	1169.166	514.767	1169.811	15.094	49/25,55/28,6144/3125
78:	1185	157	*alt gerdâniye*	519 1/4	1184.235	519.275	1184.906	15.095	2025/1024,105/53
79:	**1200**	**159**	***Gerdâniye - C***	524	**1200.000**	523.822	**1200.000**	15.094	**2/1 (Do)**

THT 2 *rast-gerdaniye* octave detail of 79 MOS 159-tET in the proposed standard of *Sipürde Ahenk*.

The general profile of "Yarman-24" variants

To continue, see THT 3:96 to discern the "*a*" and "*c*" variants from the Yarman-24[108] idea.

Since the initial two Yarman-24 variants *a* and *b* – which were conceived as rudimentary replacements for *Arel-Ezgi-Uzdilek* – are basically a list of uncomplicated ratios enveloping a temperament ordinaire core in the style of Jean Philippe Rameau[109], I shall describe mainly the *c-d-e* variants to conserve space. For a better understanding, the reader can refer to *YA24-to-Mus2.xls* and *Chronicles_Yarman24.xls*; in addition, the reader should be able to decipher the optimization intent behind variant "*d*" under *Yarman24d_equalizationOFmeantonefifth.xls* and variant "*e*" under *Yarman24E.pdf*. All of these documents are mentioned in the introduction and provided in the main link for this article[110].

A comparison of the extension to *Yarman-24c* called *Yarman-24/31c* (built by exploiting all official accidentals over every natural note) against the *Arel-Ezgi-Uzdilek* (AEU) tone-system is given in THT 4:96 and it is elaborated in FHT 5: 97.[111]

The degrees of *Yarman-24c* in its main cycle of fifths around the 12-tone modified meantone temperament core (which is altogether compatible with Baroque-to-Classical common practice) are 0-14-3-17-8-22-11-1-15-5-19-10-0. In contrast, the degrees for the 17-tone cycle (reminiscent of Urmavi and quotidian *Bağlamas* of Thrace and Anatolia) are 0-14-4-18-9-23-12-2-16-7-21-11-1-15-5-19-10-0. To expand to *Yarman-24/31c*, a subsidiary 12-tone modified meantone cycle starting on $E^{\sharp}$ (whose "home key" can otherwise be considered as $C^{\sharp}$) helps exploit all accidentals of the official *Arel-Ezgi-Uzdilek* notation over every natural as shown in FHT 5:97.

Eventually, 187/125 replaces the *G* at 695.89 c found by equally dividing the 156/125 *segâh* + 2 octaves into four parts, and 16/11 replaces G^{d}.

The mathematical optimization that leads to *Yarman-24/31d* is effectuated when the three idiosyncratic sizes of fifths (*i.e.* ≈ 696 c, ≈ 704 c, ≈ 709 c) involved in said setup, aside from the pure fifth, are equalized around their respective medians in the following way:

$$4x+8z=A=8400\,\text{c}$$

$$8x+4y=B=8490.22499567306\,\text{c}$$

(*i.e.*, 7 octaves plus a Pythagorean *leimma*)

$$5z+4y=C=6294.13499740384\,\text{c}$$

(*i.e.*, 5 octaves plus a Pythagorean minor third)

$$x=\frac{5A+8B-8C}{84}=\frac{78658254501151}{110918840956}=$$

709.151 c

$$y=\frac{10A-5B-16C}{84}=\frac{624919759933015}{887350727648}=$$

704.253 c

$$z=\frac{A-B+2C}{21}=\frac{154271311506449}{221837681912}=$$

695.424 c

Lastly, THT 5:97 below delineates a new configuration of suchlike class of fifths leading to the construction of *Yarman-24/42e*. *Yarman-24/42e* is hence an attempt to adopt into the Yarman-24/31 manifold the maximum amount of traditional perdes identified by maestro Fikret Karakaya (who is the founder and director of the "*Bezmara* Ensemble"), by allowing only one more microtonal sharp & flat pair while shifting *perde rast* back unto G in accordance with prevalent habits[112].

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[108] [Yarman, 2010a, p. 64–69] & [Yarman, 2018].

[109] [Yarman, 2010a, p. 64–99].

[110] www.ozanyarman.com/files/searchfortheoptimaltonesystem.zip.

[111] *cf.* [Yarman, 2018].

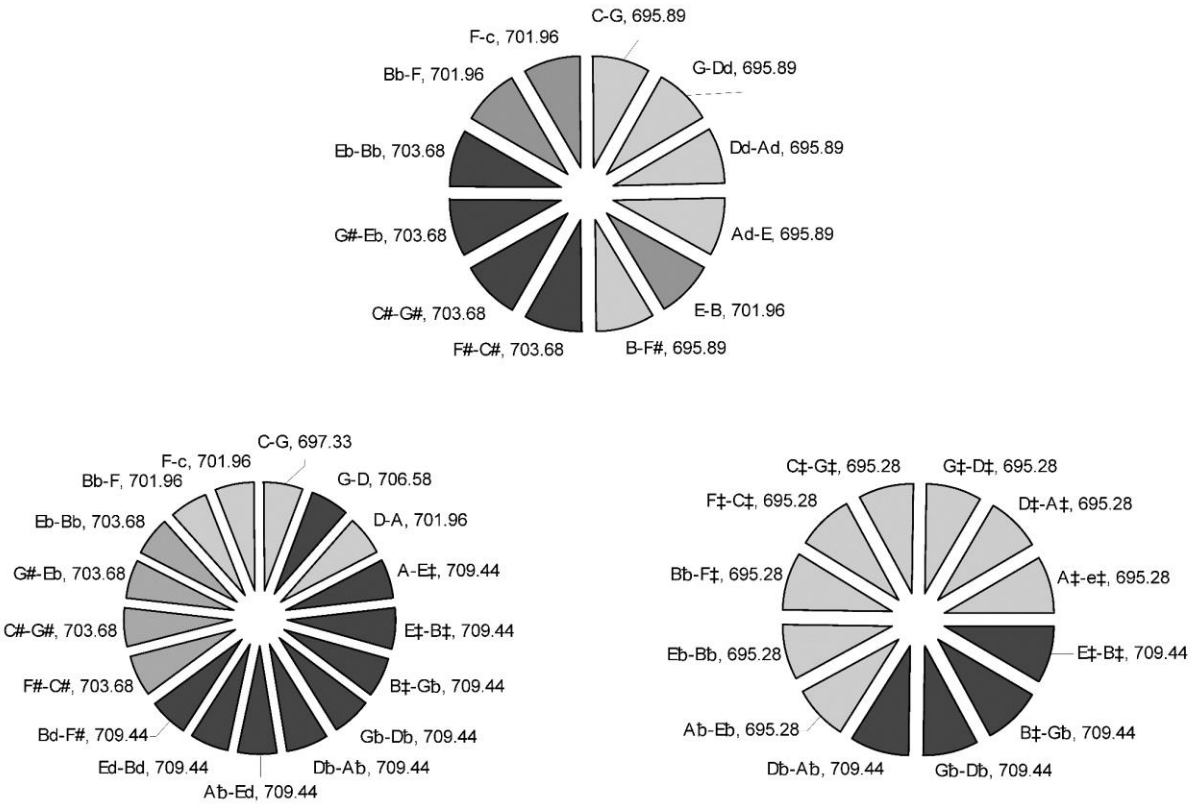
[112] *cf.* *Yarman24E.pdf* in www.ozanyarman.com/files/searchfortheoptimaltonesystem.zip.

Perde names & note	Yarman-24a	consecutive intervals	Yarman-24c	consecutive intervals	Diffs.	AEU53tET
RAST (C)	1/1 (0 cents)	*(with previous in ¢)*	1/1 (0 cents)	*(with previous in ¢)*	0	0 (*k. çargah*)
nim-zengule / şuri	84.36 cents	84.36	85.06 cents	85.06	-0.7	4 commas
zengule	38/25 (142.32)	58.01	143.62 cents	58.56	-1.3	5 "
dik-zengule	192.18 cents	49.81	191.77 cents	48.15	0.41	8 "
DÜGAH (D)	9/8 (203.91)	11.73	9/8 (203.91)	12.14	0	9 "
kürdi / nihavend	292.18 cents	88.27	292.41 cents	88.50	-0.23	13 "
dik-kürdi / uşşak	17/14 (336.13)	43.95	348.34 cents	55.93	-12.21	14 "
segah (d)	16/13 (359.47)	23.34	362.5 cents	14.16	-3.03	14 "
SEGAHÇE (E)	5/4 (386.31)	26.84	156/125 (383.54)	21.04	2.77	17 "
buselik / nişabur (ǂ)	19/15 (409.24)	22.93	415.3 cents	31.76	-6.06	18 "
ÇARGAH (F)	4/3 (498.04)	88.80	4/3 (498.04)	82.74	0	22/23 "
nim-hicaz / uzzal	584.08 cents	86.03	581.38 cents	83.34	2.7	26 "
hicaz / saba	**36/25** (631.28)	47.20	634.18 cents	52.80	-2.9	27/28 "
dik-hicaz / bestenigar	696.09 cents	64.81 (*in favor of this*)	16/11 (648.68)	61.70 (*relate to* **36/25**)	~~47.41~~	28/31 "
NEVA (G)	3/2 (701.96)	5.87 (*maybe forego it*)	187/125 (697.33)	6.070	4.63	31 "
nim-hisar / bayati	788.27 cents	86.31	788.74 cents	86.78	-0.47	35 "
hisar / hüzzam	18/11 (852.59)	64.32	853.06 cents	64.33	-0.47	36 "
dik-hisar / hisarek	888.27 cents	35.68	887.66 cents	34.59	0.61	39 "
HÜSEYNİ (A)	27/16 (905.87)	17.60	27/16 (905.87)	18.21	0	40 "
acem / nevruz	16/9 (996.09)	90.22	16/9 (996.09)	90.22	0	44 "
dik-acem / arazbar	20/11 (1035)	38.91	1043.62 cents	47.53	-8.62	45 "
evc (d)	13/7 (1071.7)	36.71	1071.94 cents	28.32	-0.24	48 "
MAHUR (B)	15/8 (1088.27)	16.57	234/125 (1085.5)	13.55	2.77	48 "
dik-mahur (ǂ)	21/11 (1119.46)	31.19	1124.74 cents	39.25	-5.28	49/52 "
GERDANİYE (c)	2/1 (1200)	80.54	2/1 (1200)	75.26	0	52/53 "

THT 3 Octave detail of *Yarman-24a versus Yarman-24c* in the proposed standard of *Sipürde Ahenk*.

Perde names & note	Arel-Ezgi-Uzdilek ratios and c	cons. intervals	53-tET subset	Yarman-24/31c	cons. intervals	Diffs.
RAST (G)	1/1 (0 cents)	*(with previous)*	0 (*rast on C*)	1/1 (0 cents)	*(with previous)*	0
---		90.22	1-2	34.18 cents	34.18	---
nim-zirgule	256/243 (90.22)		4	85.06 cents	50.88	5.16
zirgule	2187/2048 (113.69)	23.46	5-6-7	143.62 cents	58.56	-29.93
dik-zirgule	65536/59049 (180.45)	66.76	8	191.77 cents	48.15	-11.32
DÜGAH (A)	9/8 (203.91)	23.46	9	9/8 (203.91)	12.14	0
		90.22	10	224.74 cents	20.83	---
kürdi	32/27 (294.13)		13	292.41 cents	67.67	1.72
dik-kürdi	19683/16384 (317.60)	23.46	14-15	348.34 cents	55.93	-30.74
---		66.76	16-17	362.5 cents	14.16	---
segah (d)	8192/6561 (384.60)		17	156/125 (383.54)	21.04	1.06
BUSELİK (B)	81/64 (407.82)	23.46	18	415.3 cents	31.76	-7.48
dik-buselik (d)	2097152/1594323 (474.58)	66.76	21	477 cents	61.70	-2.42
ÇARGAH (C)	4/3 (498.04)	23.46	22	4/3 (498.04)	21.04	0
		90.22	23-24	538.9 cents	40.86	---
nim-hicaz	1024/729 (588.27)		26	581.38 cents	42.48	6.89
hicaz	729/512 (611.73)	23.46	27-28	634.18 cents	52.80	-22.45
dik-hicaz	262144/177147 (678.49)	66.76	29-30	16/11 (648.68)	14.50	29.81
NEVA (D)	3/2 [*440 Hz*] (701.96)	23.46	31	187/125 (697.33)	48.65	4.63
		90.22	32	729.46 cents	32.13	---
nim-hisar	128/81 (792.18)		35	788.74 cents	59.27	3.44
hisar	6561/4096 (815.64)	23.46	36-37-38	853.06 cents	64.33	-37.42
dik-hisar	32768/19683 (882.40)	66.76	39	887.66 cents	34.59	-5.26
HÜSEYNİ (E)	27/16 (905.87)	23.46	40	27/16 (905.87) *440 Hz*	18.21	0
		90.22	41	920.02 cents	14.16	---
ACEM (F)	16/9 (996.09)		44	16/9 (996.09)	76.07	0
dik-acem	59049/32768 (1019.55)	23.46	45-46	1043.62 cents	47.53	-24.07
---		66.76	47	1071.94 cents	28.32	---
eviç (#)	4096/2187 (1086.31)		48	234/125 (1085.5)	13.55	0.81
mahur	243/128 (1109.78)	23.46	49-50	1124.74 cents	39.25	-14.96
dik-mahur (d)	1048576/531441 (1176.54)	66.76	52	1186.45 cents	61.70	-9.91
GERDANİYE (g)	2/1 (1200)	23.46	53	2/1 (1200)	13.55	0

THT 4 Octave detail of AEU versus the extension to *Yarman-24c* called *Yarman-24/31c* in the proposed standard of *Sipürde Ahenk*.



FHT 5 *Yarman-24/31c* cycles of fifths (left: 17-tone cycle on *C*; middle: 12-tone “core” cycle on *C*; right: 12-tone cycle on *E* comma sharp).

Fikret Karakaya's perde name and its 8th	Ozan Yarman's interpretation	Yarman-24/42e notes & cent values	Fikret Karakaya's perde name and its 8th	Ozan Yarman's interpretation	Yarman-24/42e notes & cent values
YEGÂH – NEVÂ	YEGÂH – NEVÂ	7-RE 190.709 cents	DÜGÂH – MUHAYYER	DÜGÂH 440 – MUHAYYER	31-LA 900.000 cents
— — — — Pest bayâtî – Bayâtî Pest hisar – Hisar Pest hüzzam – Hüzzam Pest arazbar – Arazbar	—Dik Yegâh —Dik Nevâ Pest bayâtî – Bayâtî Pest nim hisar – Nim hisar Pest hisârek – Hisârek Pest hisar/hüzzam – Hisar/hüzzam Pest arazbar – Arazbar	8-Re ‡ 225.183 cents 9-Re ∦ 265.526 cents Re # / Mi b 289.731 cents 11-Mi /b 320.537 cents 12-Mi ƀ 348.410 cents 13-Mi d 362.347 cents	Dilârâ – Tiz dilârâ Dik dilârâ – Tiz dik dilârâ Kürdî – Nim sümbüle Dik Kürdî – Sümbüle Nim uşşak – Tiz nim uşşak Uşşak – Tiz uşşak Nim segâh – Tiz nim segâh	Dik Dügâh – Dik Muhayyer Alt kürdî – Alt sünbüle Kürdî (nihâvend) – Sünbüle Dik kürdî – Dik sünbüle — — Uşşak – Tiz uşşak Segâhçe – Tiz segâhçe	32-La ‡ 920.537 cents 33-La ∦ 967.481 cents 34-La # / Si b 993.888 cents 35-Si /b 1029.828 cents — *tempered to below* 36-Si ƀ 1043.765 cents 37-Si d (1057.701 cents)
Pest dik hisar – Dik hisar HÜSEYNÎ-AŞÎRAN – HÜSEYNÎ	PEST DİK HİSAR – DİK HİSAR (Hüseynî-)Aşîran – Hüseynî	14-Mİ 381.418 cents 15-Mi‡ 401.955 cents	Segâh – Tiz segâh BUSELİK – TİZ BUSELİK	SEGÂH – TİZ SEGÂH Bûselik – Tiz bûselik	38-Sİ 1076.772 cents 39-Si‡ 1111.246 cents
Pest dilâviz – Dilâviz Pest dik dilâviz – Dik dilâviz	Dik (h.) aşirân – Dik hüseynî Alt Acemaşirân – Alt Acem	16-Mi ∦ [415.892 cents] 17-Fa d [476.772 cents]	Dik buselik – Tiz dik buselik Dilkeş – Tiz dilkeş	Dik bûselik – Tiz dik bûselik Alt Çârgâh – Tiz alt Çârgâh	40-Si ∦ 1125.183 cents 41-Do d [1186.063 cents]
ACEM-AŞÎRAN – ACEM	ACEMAŞÎRAN – ACEM	18-FA 4/3 498.045 cents	**ÇÂRGÂH – TİZ ÇÂRGÂH**	**ÇÂRGÂH – TİZ ÇÂRGÂH**	**0=> 42-DO 0, 1200 cents**
Dik acem-aşîran – Dik acem Pest nevruz –Nevruz Irak – Evc Geveşt – Mahur Dik geveşt – Dik mahur	Dik Acemaşîran – Dik Acem Pest nevruz – Nevruz Irak – Eviç Geveşt/rehavî –Mahur Alt Rast – Alt Gerdâniye	19-Fa ‡ 539.119 cents 20-Fa ∦ [566.992 cents] Fa # / Sol b 580.929 cents Fa ≡ / Sol ƀ 620.537 cents 23-Sol d [648.410 cents]	Dik çârgâh – Tiz dik çârgâh Nim hicaz – Tiz nim hicaz Hicaz – Tiz hicaz Dik hicaz – Tiz dik hicaz Sabâ – Tiz sabâ Bestenigâr – Tiz bestenigâr	Dik Çârgâh – Tiz dik Çârgâh Nim uzzâl – Tiz nim uzzâl Nim hicaz – Tiz nim hicaz Uzzâl – Tiz uzzâl Hicaz/sabâ – Tiz hicaz/sabâ (Dik hicaz) – Tiz bestenigâr	1-Do ‡ 34.474 cents 2-Do ∦ 62.347 cents 3-Do # / Re b 76.284 cents 4-Re /b 115.892 cents 5-Re ƀ 129.828 cents 6-Re d [143.765 cents]
RAST – GERDANİYE	RAST – GERDÂNİYE	24-SOL 695.354 cents	NEVÂ – TİZ NEVÂ	NEVÂ – TİZ NEVÂ	7-RE 190.709 cents
— — — — Nim zengûle – Nim şehnaz Zengûle – Şehnaz Dik zengûle — — Dik şehnaz	—Dik Rast —Dik Gerdâniye —Şûrî —Tiz şûrî Nim zengûle – Nim şehnaz Alt zengûle – Tiz alt şehnaz Zengûle (şeddisabâ) – Şehnaz Dik zengûle – Dik şehnaz	25-Sol ‡ 729.828 cents 26-Sol ∦ 763.571 cents Sol # / La b 780.440 cents 28-La /b 811.246 cents 29-La ƀ 839.119 cents 30-La d (886.063 cents)	Blue color signifies that I have foregone to include the suggested perde so as to be notationally consistent.	Red color quantities signify criticality for the exploitation of all *small semitone sharps* in the 17-tone cycle as well as the subsidiary 12-tone cycle.	Green color quantities signify criticality for the exploitation of all *comma flats* to acquire the intermediary 31-tone cast, which is attained when red and gray values are omitted.
DÜGÂH – MUHAYYER	DÜGÂH [440 Hz] – MUHAYYER	31-LA 900.000 cents			

THT 5 Two octaves detail of *Yarman-24/42e* as derived from the *perde* designations of Fikret Karakaya.

Derivation of the triple 12-tone bike-chain construct named "Yarman-36"

When coming face to face with the universal penchant to eschew 53-tET (despite its unparalleled success in housing Yekta-Arel-Ezgi-Uzdilek with less than 1 cent absolute error at any degree [*cf.* Yarman 2007a]) in favor of 72-tET or similarly complex divisions of the octave in the praxis of Turkish *Makam* music, one can easily come to understand that critical "middle seconds" associated with the historical *mujannabāt* are not befittingly captured by the "Holderian comma system".

Therefore, if – in prioritizing a minimalistic approach when confronted with the inadequacy of 53-tET – one should consign to represent the relevant "middle seconds" in terms of merely two interval classes (*e.g.*, a minor wholetone and a neutral second) in practically the same vein as the Yarman-24 framework built upon a 12-tone temperament ordinaire core, one arrives at the triplex modified meantone solution dubbed *Yarman-36* [Yarman and Karaosmanoğlu, 2014] which has the advantage of being entirely tunable by ear contingent upon solely counting simple (*i.e.*, 0, 1, 2) integer beats starting from a preordained reference frequency. (*cf. Yarman36_ahenkler-PB-beats.xls* in www.ozanyarman.com/files/searchfortheoptimaltonesystem.zip.)

The triplex structure of *Yarman-36*, thusly involving a triple bike-chain (*i.e.*, three independent cycles) of 12-tone modified meantone temperaments for suitable *makam* sonority at *Sipürde*, *Bolahenk*, *Davud*, *Mansur* and *Kız Ahenks*, is shown in THT 6 below.

In concluding this appendix, the reader is directed to scrutinize THT 6:99 regarding the *perde* nomenclature under *Mus2okur* with respect to the official AEU notation (*cf.* [Yarman and Karaosmanoğlu, 2009]) and other voluminous equal divisions of the octave.

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LAYER I	C4-C5 Hertz	5th beat per sec.	LAYER II	C𝄲4-C𝄲5 Hertz	5th beat per sec.	LAYER III	C‡4-C‡5 Hertz	5th beat per sec.
+Eb	311.2375227	-2	Eb	307.3580471	-1	-Eb	301.8454475	-1
+Bb => **Bb**	465.8562841 / **2**	-1	*Bb* => **Bb**	*460.5370707* / **2**	0 (*via +C# 5:3*)	-*Bb* => **Bb**	452.2681712/ **2**	0
+F	348.8922131	-2	F	345.402803	0	-F	339.2011284	0
+c => C	522.3383196 / **2**	-1	c => C	518.1042046 / **2**	-2	-c => C	508.8016926/ **2**	-1
+G => **G**	391.2537397 / **2**	-1	G => **G**	387.5781534 / **2**	-1	-G => **G**	381.1012695/ **2**	-1
+D	292.9403048	-2	D	290.183615	-1	-D	285.3259521	-1
+*A* => **A**	*438.4104572* / **2**	-1	A => **A**	434.7754226 / **2**	-1	-A => **A**	427.4889281/ **2**	-1
+E	328.3078429	-2	E	325.5815669	-2	-E	*320.1166961*	-2 (*via +G 9:11*)
+B => **B**	491.4617643 / **2**	-1	B => **B**	487.3723504 / **2**	-1	-B => **B**	479.1750442/ **2**	-1
+F#	368.0963232	1	F#	365.0292628	-1	-F#	358.8812831	-2
+c# => C#	552.6444848 / **2**	1	c# => C#	547.0438942 / **2**	-1	-c# => C#	537.3219247/ **2**	-1
+G# => **G#**	414.9833636 / **2**	0 (*pure*)	G# => **G#**	409.7829206 / **2**	0.0417132905	-G# => **G#**	402.4914435/ **2**	-0.0462703165
+Eb	311.2375227	(-2 to Bb)	Eb	307.3580471	(-1 to Bb)	-Eb	301.8454475	(-1 to Bb)

THT 6 Derivation of *Yarman-36* by listening to integer beats, starting from a special reference frequency to get a pure fifth between *G*$^{\#}$-*E*^{b}.

53-tET deg. & c		AEU subset	Mus2okur	Perde name	72-tET	65-tET	60-tET	41-tET	34-tET	29-tET
0	**0.00**	C Dbb		KABA ÇÂRGÂH	**0**	**0**	**0**	**0**	**0**	**0**
1	22.64		C‡ Db=		16.7, 33.3	18.5, 36.9	20	29.3	35.3	41.4
2	45.28		#2		33.3, 50	36.9, 55.4	40	58.5	35.3	41.4
3	67.92		#3		66.7	73.8	60	58.5	70.6	82.8
4	90.57	C# Db		Kaba Nîm Hicâz	83.3	92.3	80, 100	87.8	70.6, 105.9	82.8
5	113.21	C≡ Dƀ		Kaba Hicâz	100, 116.7	110.8	120	117.1	105.9	124.1
6	135.85		d3		133.3	129.2, 147.7	140	146.3	141.2	124.1
7	158.49		d2		150, 166.7	147.7, 166.2	160	146.3	141.2	165.5
8	181.13	C## Dd		Kaba Dik Hicâz	183.3	184.6	180	175.6	176.5	165.5
9	**203.77**	D Ebb		YEGÂH (**220 Hz**)	**200**	**203.1**	**200**	**204.9**	**211.8**	**206.9**
10	226.42		D‡ Eb=		216.7, 233.3	221.5	220	234.1	211.8, 247.1	206.9, 248.3
11	249.06		#2		233.3, 250	240, 258.5	240	234.1, 263.4	247.1	248.3
12	271.70		#3		266.7	276.9	260	263.4	282.4	289.7
13	294.34	D# Eb		Kaba Nîm Hisâr	283.3, 300	295.4	280, 300	292.7	282.4	289.7
14	316.98	D≡ Eƀ		Kaba Hisâr	316.7	313.8	320	322.0	317.6	331.0
15	339.62		d3		333.3	332.3	340	351.2	317.6, 352.9	331.0
16	362.26		d2		350, 366.7	350.8, 369.2	360	351.2	352.9	372.4
17	384.91	D## Ed		Kaba Dik Hisâr	383.3	387.7	380	380.5	388.2	372.4
18	**407.55**	E Dx		HÜSEYNÎ AŞÎRÂN	**400**	**406.2**	**400**	**409.8**	**423.5**	**413.8**
19	430.19		E‡		416.7, 433.3	424.6	420, 440	439.0	423.5	413.8, 455.2
20	452.83		d2		450	443.1, 461.5	460	468.3	458.8	455.2
21	475.47		Fd		466.7, 483.3	461.5, 480	480	468.3	458.8	455.2
22	**498.11**	F Gbb		ACEM AŞÎRÂN	**500**	**498.5**	**500**	**497.6**	**494.1**	**496.6**
23	520.75	F‡ Gb=		Dik Acem Aşîrân	516.7, 533.3	516.9	520	526.8	529.4	537.9
24	543.40		#2		533.3, 550	535.4, 553.8	540	556.1	529.4, 564.7	537.9
25	566.04		#3		566.7	553.8, 572.3	560	556.1	564.7	579.3
26	588.68	F# Gb		Irak	583.3, 600	590.8	580	585.4	600	579.3
27	611.32	F≡ Gƀ		Geveşt	600, 616.7	609.2	600, 620	614.6	600	620.7
28	633.96		d3		633.3	627.7	640	643.9	635.3	620.7
29	656.60		d2		650, 666.7	646.2, 664.6	660	643.9	635.3, 670.6	662.1
30	679.25	F## Gd		Dik Geveşt	683.3	683.1	680	673.2	670.6	662.1
31	**701.89**	G Abb		RÂST	**700**	**701.5**	**700**	**702.4**	**705.9**	**703.4**
32	724.53		G‡ Ab=		716.7, 733.3	720	720	731.7	705.9, 741.2	703.4, 744.8
33	747.17		#2		733.3, 750	738.5, 756.9	740	761.0	741.2	744.8
34	769.81		#3		766.7	775.4	760, 780	761.0	776.5	786.2
35	792.45	G# Ab		Nîm Zirgûle	783.3, 800	793.8	800	790.2	776.5, 811.8	786.2
36	815.09	G≡ Aƀ		Zirgûle	816.7	812.3	820	819.5	811.8	827.6
37	837.74		d3		833.3	830.8	840	848.8	847.1	827.6
38	860.38		d2		850, 866.7	849.2, 867.7	860	848.8	847.1	869.0
39	883.02	G## Ad		Dik Zirgûle	883.3	886.2	880	878.0	882.4	869.0
40	**905.66**	A Bbb		DÜGÂH	**900**	**904.6**	**900**	**907.3**	**917.6**	**910.3**
41	928.30		A‡ Bb=		916.7, 933.3	923.1	920	936.6	917.6	910.3, 951.7
42	950.94		#2		950	941.5, 960	940, 960	936.6, 965.9	952.9	951.7
43	973.58		#3		966.7, 983.3	978.5	980	965.9	952.9, 988.2	951.7
44	996.23	A# Bb		Kürdî	1000	996.9	1000	995.1	988.2	993.1
45	1018.87	A≡ Bƀ		Dik Kürdî	1016.7	1015.4	1020	1024.4	1023.5	1034.5
46	1041.51		d3		1033.3, 1050	1033.8,1052.3	1040	1053.7	1058.8	1034.5
47	1064.15		d2		1066.7	1070.8	1060	1053.7	1058.8	1075.9
48	1086.79	A## Bd		Segâh	1083.3	1089.2	1080	1082.9	1094.1	1075.9
49	**1109.43**	B Ax		BÛSELİK	**1100, 1116.7**	**1107.7**	**1100, 1120**	**1112.2**	**1129.4**	**1117.2**
50	1132.08		B‡		1116.7, 1133.3	1126.2	1120, 1140	1141.5	1129.4	1117.2, 1158.6
51	1154.72		d2		1150, 1066.7	1144.6, 1163.1	1160	1141.5	1164.7	1158.6
52	1177.36	Cd		Dik Bûselik	1066.7, 1083.3	1181.5	1180	1170.7	1164.7	1158.6
53	**1200.00**	C Dbb		ÇÂRGÂH	**1200**	**1200**	**1200**	**1200**	**1200**	**1200**

THT 7 Mus2okur's 53-tET compliant *perde* and accidental usage with respect to the AEU notation, and its relation to other high-resolution equal divisions of the octave evaluated under the present study.

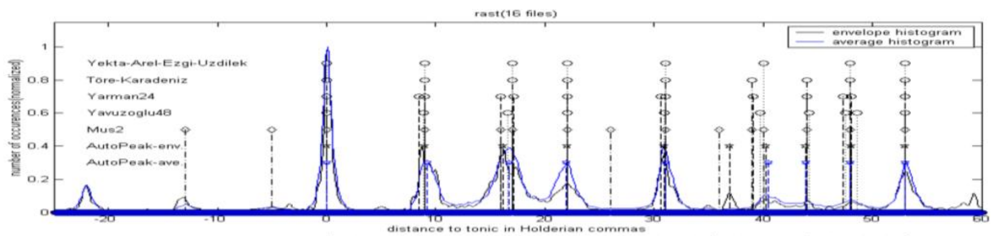
APPENDIX C

RAST	Distance to tonic in cents													M_e	D_e	M_a	D_a	E	C
79 tone-formal	196.2			377.3	392.4	498	702		898.2			1003.8	1079.3	1094.4	12.9	7.04	16.7	6.87	77.8	91.1
79 t.-matching	196.2	211.3	362.2	377.3	392.4	498	702	837.8	898.2	913.2	988.7	1003.8	1079.3	1094.4	7.5	3.52	**7.4**	**3.04**	57.1	89.9
Yarman-24a		203.9	359.5	386.3		498	702			905.9	996.1		1071.7	1088.3	5.7	2.33	9.5	3.99	77.8	**70.8**
Yarman-24b		203.9	364.7	386.3		498	702			905.9	996.1		1074.6	1088.3	**5.2**	**1.59**	9.5	3.99	77.8	**70.8**
Yarman24/31c		203.9	362.5	383.5		498	697.3			905.9	996.1		1071.9	1085.5	**5.2**	2.17	9	4.29	77.8	77.4
Yarman24/31d		203.9	360.8	381.7		498	702			905.9	996.1		1069.9	1083.7	5.8	2.8	9	3.61	77.8	77.4
Yarman24/42e		204.6	362.3	381.4		504.6	695.4			906.6		1002.7	1071.6	1085.6	7.8	4.34	8.8	5.93	77.8	83.3
Yarm36a-form	198.7			381.6		501.4	699.7		896.8				1080		16.4	8.22	18.1	7.83	**100**	83.3
Yarm36a-mtch	198.7		352.3	381.6		501.4	699.7	853.1	896.8			1001.9	1080		16.9	8.71	18.1	7.81	88.9	77.8
Yektay65-form	184.6	203.1	295.4	387.7		498.5	683.1		886.2		996.9			1181.5	24.9	7.57	28.7	7.19	77.8	89.2
Yektay65-mtch		203.1	369.2	387.7		498.5	701.5	830.8		904.6	996.9			1089.2	6.5	2.73	10.3	4.4	88.9	87.7
Durgun60-mtch		200	360	380		500	700	840		900		1000	1080		11.1	5.18	14.9	6.43	88.9	88.9
72 tET-matching		200	366.7	383.3		500	700	833.3		900		1000		1083.3	11.1	4.19	14.9	6.43	77.8	91.1
41 tET-matching		204.9	351.2	380.5		497.6	702.4	848.8		907.3	995.1		1082.9		14	5.22	7.6	**2.99**	88.9	80.5
34 tET-matching		211.8	352.9	388.2		494.1	705.9	847.1		917.6	988.2			1094.1	12.3	7.44	11.4	5.24	88.9	76.5
29 tET-matching		206.9		372.4		496.6	703.4	827.6		910.3	993.1		1075.9		13.6	5.07	11.1	3.4	**100**	72.4
AutoPeak-env.	203.8		365.2			497.9	699.2	836.2	911.1		994.9		1089.5							
AutoPeak-ave.	207.6		376.8			495.8	702.3		914.9		994.2		1087.0							

Continued table: Complexity-weighted (WM, WD) and Efficiency-scaled (WEM, WED) means of M's & D's

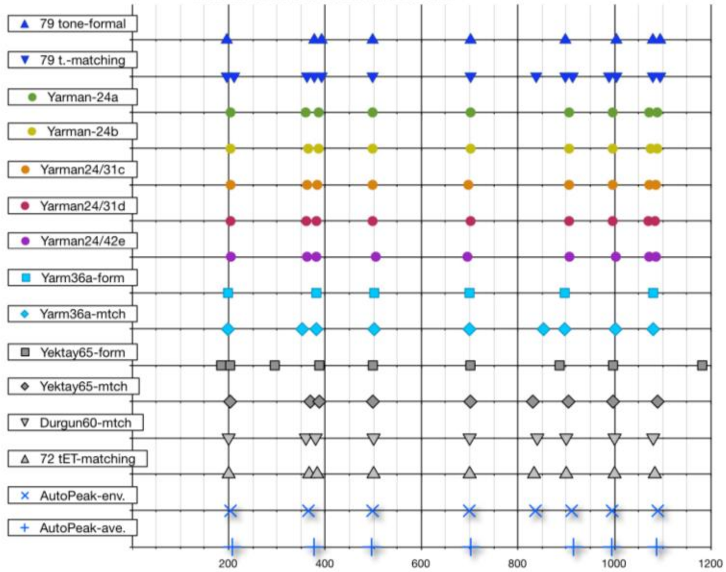
41 tET-matching	**WM:**	11.84	**WD:**	4.5	**WEM:**	13.32	**WED:**	5.06
34 tET-matching	**WM:**	12.52	**WD:**	6.7	**WEM:**	14.08	**WED:**	7.53
29 tET-matching	**WM:**	12.55	**WD:**	4.3	**WEM:**	12.55	**WED:**	4.3

THT 8 Mismatches in cents of the scale tones of various tunings with *RAST* auto-peaks (16 collated histograms) and the resultant *efficiency* (E) and *complexity* (C) percentages. "M" denotes the maximum difference and "D" denotes the mean of differences, with subscript "e" delineating values in reference to *Auto-Peak envelope* datapoints and subscript "a" delineating values in reference to *Auto-Peak average* datapoints, where best values are highlighted and in bold (while red, blue and purple text colors are cosmetic). Higher E and lower C are better.



WM	WD	WEM	WED
17.81	8.37	22.89	10.76
8.87	3.91	15.52	6.84
7.6	3.16	9.77	4.06
7.35	**2.79**	**9.45**	**3.58**
7.57	3.44	9.73	4.42
7.89	3.42	10.14	4.4
9.34	5.78	12.01	7.43
19.41	9.03	19.41	9.03
18.72	8.84	21.05	9.94
31.73	8.74	40.8	11.23
9.82	4.16	11.04	4.68
15.06	6.72	16.94	7.56
15.35	6.27	17.27	7.05

Continued table: Complexity and Efficiency scaled means of M's & D's



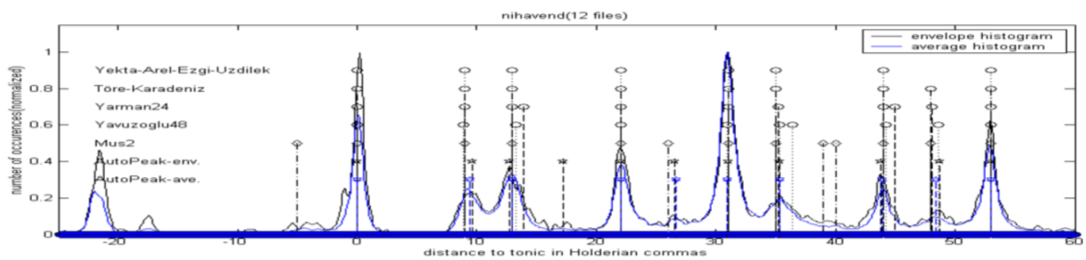
FHT 6 Plotting the data of THT 8 in comparison with the relevant histogram graphic in [Bozkurt et al., 2009].

NİHAVEND	Distance to tonic in cents														M_e	D_e	M_a	D_a	E	C
79 tone-formal	196.2			316.9			498		702		822.7		1003.8	1079.3	1094.4	28.4	13.17	24.8	10.93	87.5	91.1
79 t.-matching	196.2	211.3	286.8	301.8	316.9	392.4	498	603.7	702	807.6	822.7	988.7	1003.8	1079.3	1094.4	**7.2**	3.56	8.1	**3.06**	60	88.6
Yarman-24a	203.9		292.2				498		702	788.4		996.1		1088.3		13	6.84	11.1	4.34	**100**	70.8
Yarman-24b	203.9		292.2				498		702	788.3		996.1		1088.3		13	6.86	11.2	4.36	**100**	70.8
Yarman24/31c	203.9		292.4				498		697.3	788.7		996.1		1085.5		15.6	7.23	10.8	4.73	**100**	77.4
Yarman24/31d	203.9		291.8				498		702	787.6		996.1		1083.7		17.4	7.56	11.9	5.14	**100**	77.4
Yarman24/42e	204.6		298.5				504.6		695.4	794.4			1002.7	1085.6		15.5	9.43	9.3	6.67	**100**	83.3
Yarm36a-form	198.7			303.6			501.4		699.7	801.7			1001.9		1094.5	18.2	7.87	13.7	5.37	**100**	80.6
Yarm36a-mtch	198.7		281.9	303.6		396.1	501.4		699.7	801.7			1001.9	1080	1094.5	18.2	6.49	13.7	5.19	80	77.8
Yektay65-form	203.1		295.4				498.5		701.5	793.8	867.7	886.2	996.9	1089.2		13.8	6.6	9.3	3.99	77.8	89.2
Yektay65-mtch	203.1	221.5	295.4			387.7	498.5	609.2	701.5	793.8		996.9		1089.2	1107.7	9.7	4.94	9.1	4.1	81.8	86.2
Durgun60-mtch	200	220	280	300		400	500	600	700	800			1000		1100	9.2	3.68	**7.9**	3.92	81.8	85
72 tET-matching	200	216.7	283.3	300		400	500	600	700	800			1000		1100	9.2	**2.99**	**7.9**	3.51	81.8	87.5
41 tET-matching		204.9	292.7			380.5	497.6	614.6	702.4	790.2		995.1		1082.9	1112.2	15.1	7.68	12	5.47	90	78
34 tET-matching		211.8	282.4			388.2	494.1	600	705.9		811.8	988.2			1094.1	11.4	5.11	12.3	5.61	**100**	73.5
29 tET-matching		206.9	289.7			372.4	496.6	620.7	703.4	786.2		993.1		1075.9	1117.2	21.2	9.7	19	8.04	90	**69**
AutoPeak-env.	216.9		288.5			390.8	499.9	599.5	701.9	800.4		991.7		1101.1							
AutoPeak-ave.	212.4		292.1				499.7	604.1	700.8	799.5		994.9		1094.9							

Continued table: Complexity-weighted (WM, WD) and Efficiency-scaled (WEM, WED) means of M's & D's

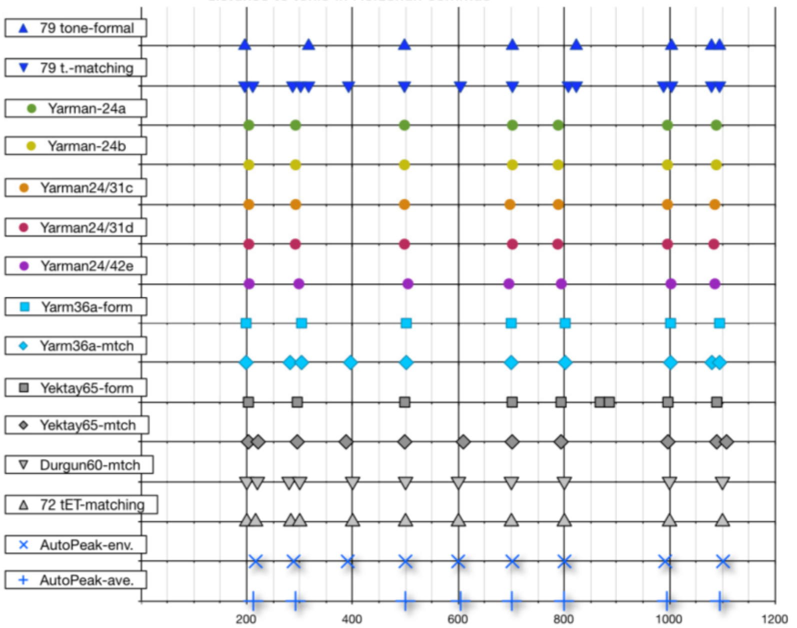
41 tET-matching	**WM:** 14.78	**WD:** 7.17	**WEM:** 16.42	**WED:** 7.97
34 tET-matching	**WM:** 12.39	**WD:** 5.61	**WEM:** 12.39	**WED:** 5.61
29 tET-matching	**WM:** 20.1	**WD:** 8.87	**WEM:** 22.33	**WED:** 9.85

THT 9 Mismatches in cents of the scale tones of various tunings with *NIHAVEND* auto-peaks (12 collated histograms) and the resultant *efficiency* (E) and *complexity* (C) percentages. "M" denotes the maximum difference and "D" denotes the mean of differences, with subscript "e" delineating values in reference to *Auto-Peak envelope* datapoints and subscript "a" delineating values in reference to *Auto-Peak average* datapoints, where best values are highlighted and in bold (while red, blue and purple text colors are cosmetic). Higher E and lower C are better.



32.5	14.72	37.14	16.83
9.15	3.96	15.25	6.6
12.28	5.7	12.28	5.7
12.33	5.71	12.33	5.71
14.32	7.01	14.32	7.01
15.89	6.89	15.89	6.89
14.18	9.21	14.18	9.21
17.8	7.39	17.8	7.39
17.36	6.35	21.69	7.94
13.89	6.37	17.86	8.18
11.02	5.3	13.46	6.48
9.92	4.41	**12.13**	5.39
10.13	**3.85**	12.39	**4.71**
WM	WD	WEM	WED

Continued table: Complexity and Efficiency scaled means of M's & D's



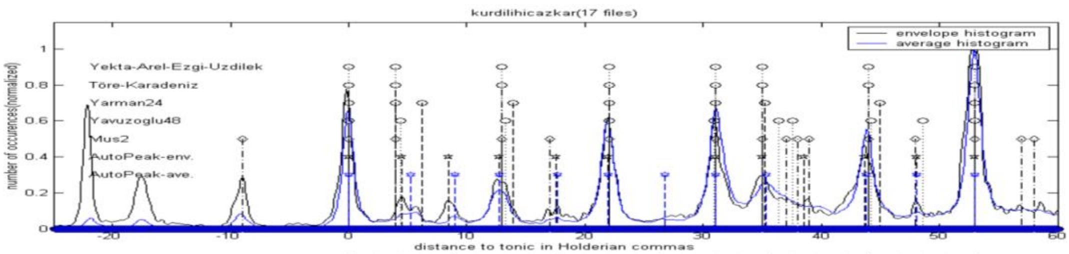
FHT 7 Plotting the data of THT 9 in comparison with the relevant histogram graphic in [Bozkurt et al., 2009].

K.HİCAZKAR	Distance to tonic in cents														M_e	D_e	M_a	D_a	E	C
79 tone-formal	105.6	120.7		286.8	392.4	498		702	807.6	822.7				1003.8	1094.4	15.6	5.51	12.8	5.06	90	88.6
79 t.-matching	105.6	120.7	196.2	286.8	392.4	498	603.7	702	792.5		868		988.7		1094.4	**6.2**	**2.38**	7.9	4	**100**	84.8
Yarman-24a	84.4	142.4		292.2	386.3	498		702	788.3					996.1	1088.3	17.9	5.09	23.3	7.84	**100**	62.5
Yarman-24b	84.4	145.1		292.2	386.3	498		702	788.3					996.1	1088.3	17.9	5.09	26	8.18	**100**	62.5
Yarman24/31c	85.1	143.6		292.4	383.5	498		697.3	788.7					996.1	1085.5	17.2	6.11	24.5	9.16	**100**	71
Yarman24/31d	83.3	142.5		291.8	381.7	498		702	787.6					996.1	1083.7	19	6.24	23.4	8.94	**100**	71
Yarman24/42e	85.1	115.9		298.5	381.4	504.6		695.4	794.4	825.2				1002.7	1085.6	16.6	8.19	18	8.68	90	78.6
Yarm36a-form	97.6			303.6		501.4		699.7	801.7					1001.9		14.7	8.28	21.5	9.93	**100**	83.3
Yarm36a-mtch	97.6	153.2	198.7	303.6	381.6	501.4	594.1	699.7	801.7			882.3	982	1001.9	1080	16.4	7.02	21.5	9.98	92.3	66.7
Yektay65-form	92.3	110.8		295.4	387.7	498.5		701.5	793.8		867.7	886.2		996.9		10.3	4.79	11.7	6.49	90	86.2
Yektay65-mtch		110.8	203.1	295.4	406.2	498.5	609.2	701.5	793.8		867.7	886.2		996.9	1089.2	11.1	4.42	8.3	4.63	**100**	81.5
Durgun60-mtch	100	120	200	280	400	500	600	700	800			880	980	1000	1080	8.9	4.93	9	4.42	92.3	80
72 tET-matching	100	116.7	200	283.3	400	500	600	700	800			883.3	983.3	1000	1083.3	10.5	4.38	**7.7**	**3.78**	92.3	83.3
41 tET-matching	87.8	117.1	204.9	292.7	409.8	497.6	614.6	702.4	790.2		878		995.1		1082.9	14.5	5.23	10.4	5.04	**100**	70.7
34 tET-matching	105.9		211.8	282.4	388.2	494.1	600	705.9		811.8		882.4	988.2		1094.1	19.8	7.42	13.2	6.71	**100**	67.6
29 tET-matching	82.8	124.1	206.9	289.7	413.8	496.6	620.7	703.4	786.2		869		993.1		1075.9	19.5	6.53	15.27	6.95	**100**	**58.6**
AutoPeak-env.	102.3	192		288.9	398.0	496.5		702.3	792		872.8		988.8		1088.2						
AutoPeak-ave.	119.1		203.8	287.8	399.4	495.8	605.434	704.1509	800.4				991		1087.9						

Continued table: Complexity-weighted (WM, WD) and Efficiency-scaled (WEM, WED) means of M's & D's

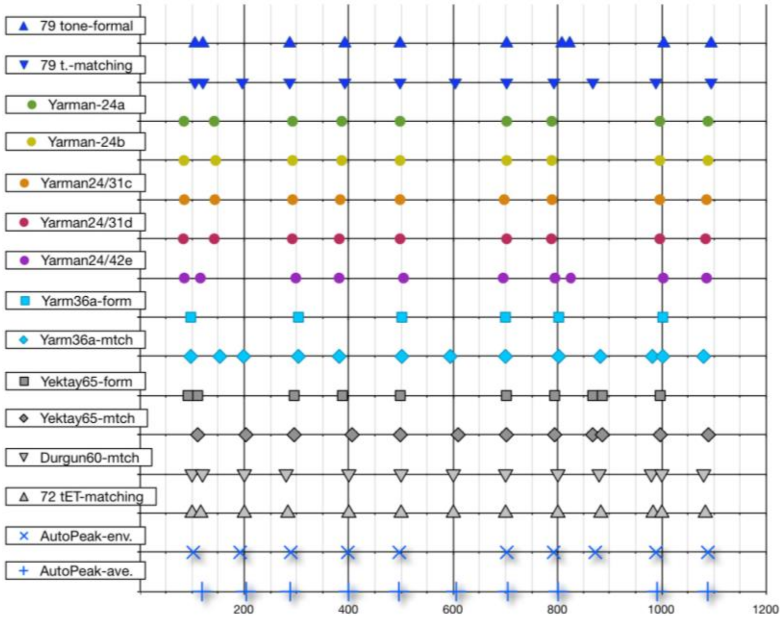
41 tET-matching	**WM:** 13.96	**WD:** 5.76	**WEM:** 13.96	**WED:** 5.76
34 tET-matching	**WM:** 17.99	**WD:** 7.7	**WEM:** 17.99	**WED:** 7.7
29 tET-matching	**WM:** 17.38	**WD:** 6.74	**WEM:** 17.38	**WED:** 6.74

THT 10 Mismatches in cents of the scale tones of various tunings with *KÜRDILIHICAZKAR* auto-peaks (17 collated histograms) and the resultant *efficiency* (E) and *complexity* (C) percentages. "M" denotes the maximum difference and "D" denotes the mean of differences, with subscript "e" delineating values in reference to *Auto-Peak envelope* datapoints and subscript "a" delineating values in reference to *Auto-Peak average* datapoints, where best values are highlighted and in bold (while red, etc… text colors are cosmetic). Higher E and lower C are better.



18.46	6.87	20.51	7.63
8.9	**4.03**	**8.9**	**4.03**
21.4	6.72	21.4	6.72
22.8	6.89	22.8	6.89
23.42	8.58	23.42	8.58
23.82	8.53	23.82	8.53
20.75	10.12	23.06	11.24
22.57	11.35	22.57	11.35
20.47	9.18	22.18	9.95
14.03	7.19	15.59	7.99
11.92	5.57	11.92	5.57
10.86	5.68	11.77	6.15
11.35	5.09	12.29	5.51
WM	WD	WEM	WED

Continued table: Complexity and Efficiency scaled means of M's & D's



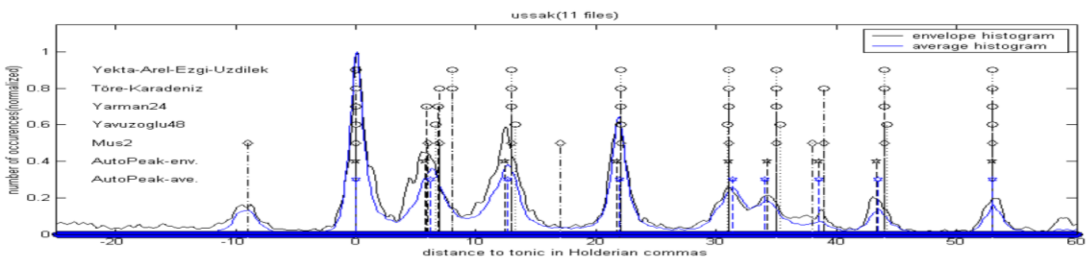
FHT 8 Plotting the data of THT 10 in comparison with the relevant histogram graphic in [Bozkurt et al., 2009].

UŞŞAK	Distance to tonic in cents															M_e	D_e	M_a	D_a	E	C
79 tone-formal	150.9	166		286.8		498		702		807.6					1003.8	31.9	14.53	38	13.12	85.7	92.4
79 t.-matching	135.8	150.9		286.8	301.8	498		702	777.4	792.5	807.6	868		988.7	1003.8	**7.5**	4.43	**8**	5.07	58.3	91.1
Yarman-24a	132.2	155.6			294.1	492.2		702		792.2		867.8			996.1	16.5	7.59	22.6	9.97	**87.5**	**70.8**
Yarman-24b	139	160.8			294.1	492.2		702		792.2		870.6			996.1	16.5	8.16	22.6	8.6	**87.5**	**70.8**
Yarman24/31c	144.1	158.6		273.1	294.1	493.4		702		792.2		868		982.5	996.1	16.5	6.49	22.6	6.9	70	77.4
Yarman24/31d	143.1	156.9		273.2	294.1	498		702		792.2		866		982.4	996.1	16.5	7.26	22.6	7.09	70	77.4
Yarman24/42e	129.8	143.8		286.1	300	490.7		702	776.8		798	867			995.4	14.2	**4.13**	12.1	5.66	70	83.3
Yarm36a-form		153.6			302.6	501		698			803.1				1001.3	23.9	15.66	18	12.74	83.3	86.1
Yarm36a-mtch		153.6		285.2	302.6	501		698		783.3	803.1	881.3	895.8	987.2	1001.3	23.9	9.07	13.7	7.97	63.6	80.6
Yektay65-form	110.8	147.7	184.6		295.4	498.5	683.1	701.5		793.8					996.9	18.1	12.15	24.2	10.73	66.7	90.8
Yektay65-mtch	129.2	147.7		276.9	295.4	498.5		701.5	775.4	793.8		867.7	886.2		996.9	15.7	4.54	13.6	7.11	63.6	89.2
Durgun60-mtch	120	140		280		500		700	760	780		880		980		9.7	4.81	10	6.11	77.8	88.3
72 tET-matching	133.3	150		283.3		500		700	766.7	783.3		866.7		983.3		7.6	4.4	10	**4.74**	77.8	90.3
41 tET-matching	117.1	146.3		292.7		497.6		702.4	761	790.2		878			995.1	14.5	10.57	11.8	7.78	66.7	85.4
34 tET-matching		141.2		282.4		494.1		705.9	776.5				882.4	988.2		11.5	5.83	10	5.28	85.7	82.4
29 tET-matching	124.1	165.5		289.7		496.6		703.4		786.2		869			993.1	11.9	7.33	18.1	9.92	75	79.3
AutoPeak-env.	129.7			281		493.1	702.8		775.7			872.2		981.2							
AutoPeak-ave.	142.2			285.7		495.6	710.0		769.6			872.4		983.3							

Continued table: Complexity-weighted (WM, WD) and Efficiency-scaled (WEM, WED) means of M's & D's

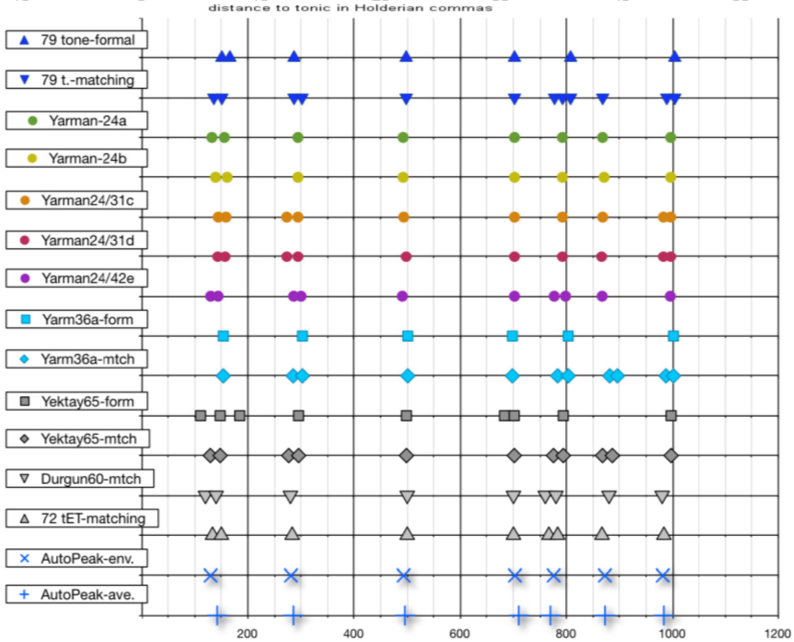
41 tET-matching	**WM:** 15.06	**WD:** 10.51	**WEM:** 22.59	**WED:** 15.76
34 tET-matching	**WM:** 11.99	**WD:** 6.2	**WEM:** 13.99	**WED:** 7.23
29 tET-matching	**WM:** 16.27	**WD:** 9.36	**WEM:** 21.7	**WED:** 12.47

THT 11 Mismatches in cents of the scale tones of various tunings with *UŞŞAK* auto-peaks (11 collated histograms) and the resultant *efficiency* (E) and *complexity* (C) percentages. "M" denotes the maximum difference and "D" denotes the mean of differences, with subscript "e" delineating values in reference to *Auto-Peak envelope* datapoints and subscript "a" delineating values in reference to *Auto-Peak average* datapoints, where best values are highlighted and in bold (while red, blue and purple text colors are cosmetic). Higher E and lower C are better.



WM	WD	WEM	WED
42.49	16.81	49.57	19.61
9.32	5.71	15.98	9.8
19.55	8.78	22.34	10.03
19.55	8.38	22.34	9.58
20.84	7.13	29.77	10.19
20.84	7.64	29.77	10.92
14.79	5.5	21.13	7.86
24.15	16.37	28.98	19.64
20.63	9.35	32.42	14.69
25.37	13.72	38.05	20.58
17.35	6.9	27.26	10.84
11.57	6.42	14.88	8.25
10.51	**5.46**	**13.51**	**7.02**

Continued table: Complexity and Efficiency scaled means of M's & D's



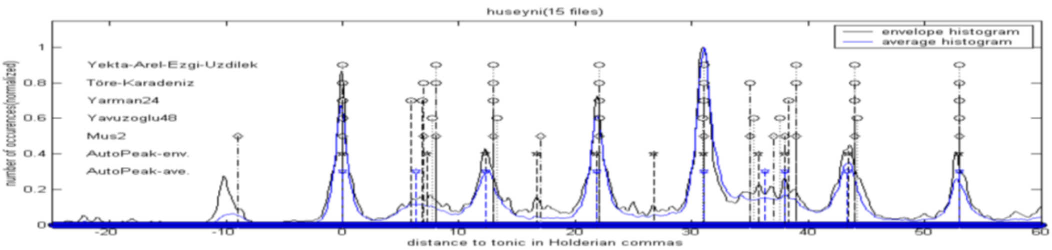
FHT 9 Plotting the data of THT 11 in comparison with the relevant histogram graphic in [Bozkurt et al., 2009].

HÜSEYNİ	Distance to tonic in cents															M_e	D_e	M_a	D_a	E	C
79 tone-formal		166		301.8			498		702			868			1003.8	22.6	9.6	22.2	13.23	**100**	92.4
79 t.-matching	150.9	166	271.1	286.8	301.8	377.3	498	603.7	702	807.6	822.7	852.9	868	988.7	1003.8	**7.6**	3.59	7.2	4.74	60	88.6
Yarman-24a	155.6			294.1			498		702	792.2		867.8			996.1	16.3	9.47	30.6	12	**100**	70.8
Yarman-24b		160.8		294.1			498		702	788.3		870.6			996.1	20.2	9.69	34.5	13.7	**100**	70.8
Yarman24/31c	144.4	158.6	273.1	294.1			493.4		702	792.2		868			996.1	16.3	7.27	30.6	8.74	77.8	77.4
Yarman24/31d	143.1	156.9	273.2	294.1			498		702	792.2		866			996.1	16.3	7.76	30.6	9.11	77.8	77.4
Yarman24/42e	143.8	157.7		286.1			490.7		702	798	839.1	867			995.4	13.2	6.83	16.3	6.87	77.8	83.3
Yarm36a-form	153.6	182.9		302.6			501		698			851.9	881.3		1001.3	23.4	12.17	23	12.03	75	83.3
Yarm36a-mtch	153.6	182.9		285.2	302.6	380.9	501	602.9	698	803.1		851.9	881.3	987.2	1001.3	11.2	5.76	19.7	8.63	69.2	75
Yektay65-form	147.7	184.6	203.1	295.4			498.5		701.5	793.8	812.3		886.2		996.9	25.6	11.74	26.3	11.07	70	89.2
Yektay65-mtch	147.7	166.2	276.9	295.4		387.7	498.5	609.2	701.5	793.8	812.3	867.7	886.2	978.5	996.9	8.7	3.9	10.5	4.84	64.3	86.2
Durgun60-mtch	140	160	280	300		380	500	600	700	800	820	860		980	1000	8.5	**3.49**	**6.2**	**2.63**	69.2	85
72 tET-matching	150	166.7	283.3	300		383.3	500	600	700	800	816.7	866.7	883.3	983.3	1000	8.2	4.36	6.8	4.74	64.3	87.5
41 tET-matching	146.3	175.6		292.7		380.5	497.6	614.6	702.4		819.5	878			995.1	17.4	8.96	18.1	7.77	90	78
34 tET-matching	141.2	176.5	282.4			388.2	494.1	600	705.9		811.8	847.1		988.2		13.5	6.4	12.8	5.61	90	73.5
29 tET-matching		165.5		289.7		372.4	496.6	620.7	703.4		827.6	869			993.1	19.1	8.42	21.3	8.56	**100**	**69**
AutoPeak-env.	164.8		279.2			379	493.8	605.9	701.4	808.5		860.6		982.2							
AutoPeak-ave.	144.2		279.6				493.8		703	822.8		859.9		981.7							

Continued table: Complexity-weighted (WM, WD) and Efficiency-scaled (WEM, WED) means of M's & D's

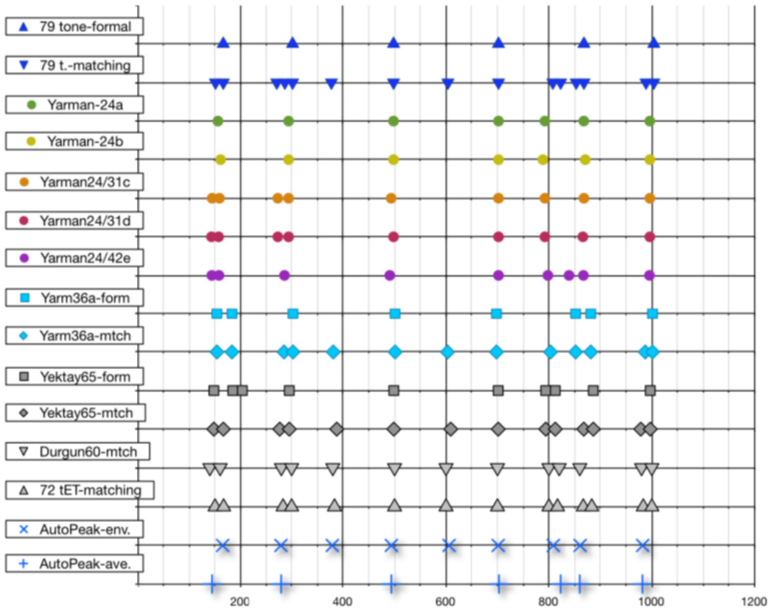
41 tET-matching	**WM:** 19.36	**WD:** 9.12	**WEM:** 21.51	**WED:** 10.14
34 tET-matching	**WM:** 13.75	**WD:** 6.28	**WEM:** 15.28	**WED:** 6.98
29 tET-matching	**WM:** 20.2	**WD:** 8.49	**WEM:** 20.2	**WED:** 8.49

THT 12 Mismatches in cents of the scale tones of various tunings with *HÜSEYNI* auto-peaks (15 collated histograms) and the resultant *efficiency* (E) and *complexity* (C) percentages. "M" denotes the maximum difference and "D" denotes the mean of differences, with subscript "e" delineating values in reference to *Auto-Peak envelope* datapoints and subscript "a" delineating values in reference to *Auto-Peak average* datapoints, where best values are highlighted and in bold (while red, blue and purple text colors are cosmetic). Higher E and lower C are better.



27.65	14.09	27.65	14.09
8.85	4.98	14.76	8.31
23.89	10.94	23.89	10.94
27.86	11.91	27.86	11.91
25.43	8.68	32.7	11.17
25.43	9.15	32.7	11.76
16.87	7.83	21.69	10.07
26.53	13.84	35.38	18.45
16.38	7.63	23.66	11.02
31.21	13.72	44.58	19.6
11.25	5.12	17.5	7.97
8.53	**3.55**	**12.32**	**5.13**
8.89	5.39	13.83	8.39
WM	WD	WEM	WED

Continued table: Complexity and Efficiency scaled means of M's & D's



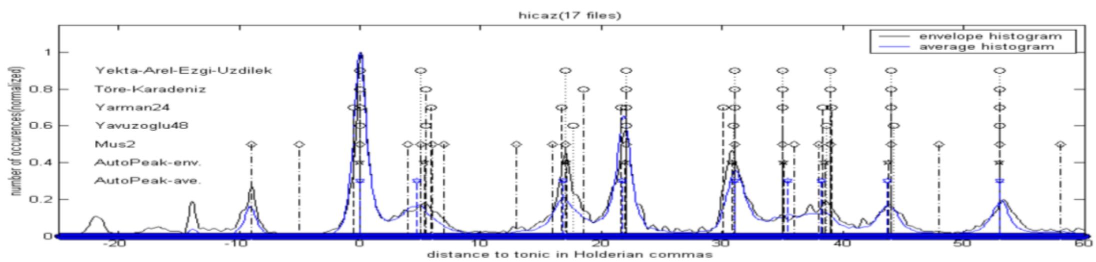
FHT 10 Plotting the data of THT 12 in comparison with the relevant histogram graphic in [Bozkurt et al., 2009].

HİCAZ	Distance to tonic in cents														M_e	D_e	M_a	D_a	E	C
79 tone-formal			120.7		392.4	498		702	777.4		868			1003.8	16.2	6.77	23.7	10.6	**100**	91.1
79 t.-matching		105.6	120.7	377.3	392.4	498		702	792.5	807.6	868		988.7	1003.8	6.6	**3**	**6.5**	**2.81**	63.6	91.1
Yarman-24a			132.2	380.2		498		702	792.2		867.8		996.1		12.2	5.09	26.9	7.37	**100**	**70.8**
Yarman-24b			139	380.4		498		702	792.2		870.6		996.1		19	5.63	33.7	8.71	**100**	**70.8**
Yarman24/31c			144.4	377.5		493.4		702	792.2		868	881.6	996.1		24.4	7.36	39.1	8.87	87.5	77.4
Yarman24/31d			143.1	375.2		498		702	792.2		866	879.7	996.1		23.1	7.49	37.8	9.39	87.5	77.4
Yarman24/42e		129.8	143.8	376.3		490.7		702	798		867	880.9	995.4		9.8	6.3	24.5	6.27	77.8	83.3
Yarm36a-form		104.9		380.9		501		698		803.1		881.3		1001.3	15.1	7.6	15.7	6.26	**100**	80.6
Yarm36a-mtch		104.9		380.9		501		698		803.1	851.9	881.3		1001.3	15.1	7.6	13.7	5.97	87.5	80.6
Yektay65-form	92.3	110.8	129.2		387.7	498.5	683.1	701.5	793.8	812.3		886.2	996.9		12.9	5.13	20.6	7.93	63.6	89.2
Yektay65-mtch		110.8	129.2		387.7	498.5		701.5	793.8		867.7	886.2	996.9		9.2	4.09	7.5	5.29	77.8	89.2
Durgun60-mtch		100	120	380		500		700		800	860	880	980	1000	9.4	4.81	9.4	4.64	70	88.3
72 tET-matching		100	116.7		383.3	500		700		800	866.7			1000	9.4	4.8	10.6	4.47	87.5	90.3
41 tET-matching			117.1	380.5		497.6		702.4	790.2			878	995.1		**5.3**	4.37	12.4	7.77	85.7	85.4
34 tET-matching		105.9			388.2	494.1		705.9		811.8	847.1	882.4	988.2		18.2	9.33	16.8	6.6	75	82.4
29 tET-matching			124.1	372.4		496.6		703.4	786.2		869		993.1		13.4	6.18	18.8	8.78	85.7	79.3
AutoPeak-env.		120		385.8		496.3	698.3		793.6		873.3		990.6							
AutoPeak-ave.		105.3		380.6		493.4	703.9		801.1		865.6		989.4							

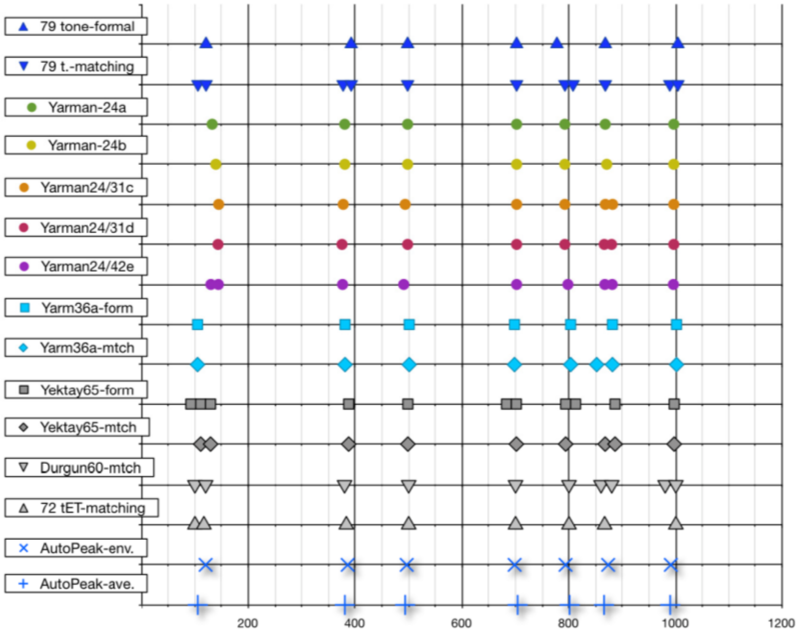
Continued table: Complexity-weighted (WM, WD) and Efficiency-scaled (WEM, WED) means of M's & D's

41 tET-matching	**WM:** 10.14	**WD:** 6.95	**WEM:** **11.83**	**WED:** 8.11
34 tET-matching	**WM:** 19.52	**WD:** 8.88	**WEM:** 26.02	**WED:** 11.85
29 tET-matching	**WM:** 17.46	**WD:** 8.12	**WEM:** 20.38	**WED:** 9.47

THT 13 Mismatches in cents of the scale tones of various tunings with *HICAZ* auto-peaks (17 collated histograms) and the resultant *efficiency* (E) and *complexity* (C) percentages. "M" denotes the maximum difference and "D" denotes the mean of differences, with subscript "e" delineating values in reference to *Auto-Peak envelope* datapoints and subscript "a" delineating values in reference to *Auto-Peak average* datapoints, where best values are highlighted and in bold (while red, blue and purple text colors are cosmetic). Higher E and lower C are better.



24	10.45	24	10.45
7.88	**3.5**	12.38	**5.5**
19.55	6.23	19.55	6.23
26.35	7.17	26.35	7.17
33.84	8.65	38.68	9.88
32.46	8.99	37.09	10.28
19.29	7.07	24.81	9.09
16.9	7.6	16.9	7.6
15.8	7.45	18.06	8.51
19.83	7.73	31.16	12.15
9.89	5.55	12.71	7.13
11.05	5.56	15.78	7.94
11.94	5.54	13.65	6.33
WM	**WD**	**WEM**	**WED**
Continued table: Complexity and Efficiency scaled means of M's & D's			



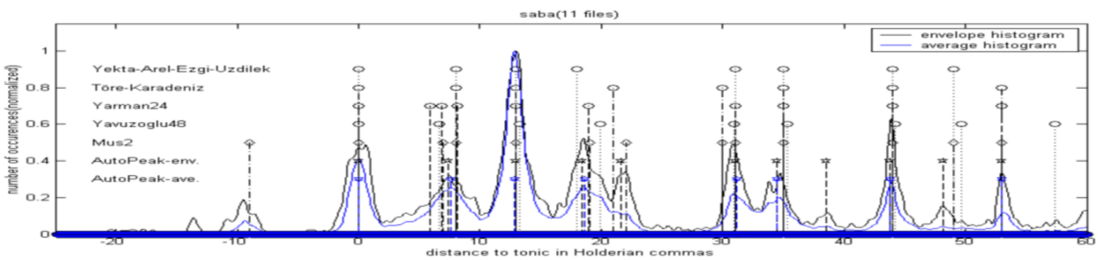
FHT 11 Plotting the data of THT 13 in comparison with the relevant histogram graphic in [Bozkurt et al., 2009].

SABA	Distance to tonic in cents																	M_e	D_e	M_a	D_a	E	C
79 tone-formal		150.9	166		301.8		437.7	452.8		702		807.6			1003.8		1154.7	25.8	9.93	25.6	12.23	77.8	91.1
79 t.-matching		150.9	166	286.8	301.8		422.6			702	777.4	792.5		988.7	1003.8	1139.6	1154.7	**5.5**	**3.01**	**6.3**	**3.47**	58.3	91.1
Yarman-24a		155.6	182.4		294.1		427.4		492.2	702		792.2			996.1	1080.5	1138.4	11.7	6.24	10.2	6.02	80	**66.7**
Yarman-24b		160.8	182.4		294.1		431.4		492.2	702		792.2			996.1	1080.5	1141.2	14.3	6.09	10.2	6.68	80	**66.7**
Yarman24/31c		158.6	179.6		294.1		430.3		493.4	702		792.2			996.1	1081.1	1139.7	13.2	6.3	10.2	6.03	80	74.2
Yarman24/31d		156.9	177.8		294.1		429.4		491.5	702		792.2			996.1	1079.4	1138.6	12.3	6.38	10.2	5.58	80	74.2
Yarman24/42e		157.7	176.8	286.1	300	415.9	429.8		490.7	702	776.8	798	867		995.4	1080.4	1139.1	9.6	4.33	6.4	4.37	64.3	78.6
Yarm36a-form		153.6	182.9	285.2	302.6	395.4	455.5			698		803.1			1001.3		1154.4	21.7	10.71	28	13.43	70	80.6
Yarm36a-mtch		153.6	182.9	285.2	302.6	395.4	455.5		484.7	698	783.3	803.1	881.3	987.2	1001.3	1098.9	1154.4	21.7	8.54	28	9.35	60	75
Yektay65-form	110.8	147.7	184.6	295.4	387.7	406.2	424.6	443.1	480	701.5		793.8			996.9			17.3	7.91	12.3	6.62	58.3	89.2
Yektay65-mtch		147.7	166.2		295.4	406.2	424.6		498.5	701.5	775.4	793.8	867.7		996.9	1089.2		7.9	3.94	6.6	4.72	75	86.2
Durgun60-mtch		160	180	280	300	400	420		500	700	780		880		1000	1080		10	6.28	9.6	5.98	75	85
72 tET-matching		166.7	183.3	283.3	300	400	416.7		483.3	700	783.3		866.7		1000	1083.3		8.2	4.21	8.1	5.52	75	87.5
41 tET-matching		146.3	175.6		292.7	409.8		439	497.6	702.4		790.2	878		995.1	1082.9		8.4	5.44	13.6	5.68	81.8	78
34 tET-matching		141.2	176.5	282.4			423.5		494.1	705.9	776.5		882.4	988.2			1094.1	10.9	6.46	7.2	3.58	90	73.5
29 tET-matching		165.5		289.7		413.8			496.6	703.4	786.2		869		993.1	1075.9		14.1	4.08	9.6	3.93	**100**	69
AutoPeak-env.		167.3		291.8		417.1			490.6	701.2	781.8		871.5	992.8		1090							
AutoPeak-ave.		172.3		289.6		423.4				705.1	782.0			991.9									

Continued table: Complexity-weighted (WM, WD) and Efficiency-scaled (WEM, WED) means of M's & D's

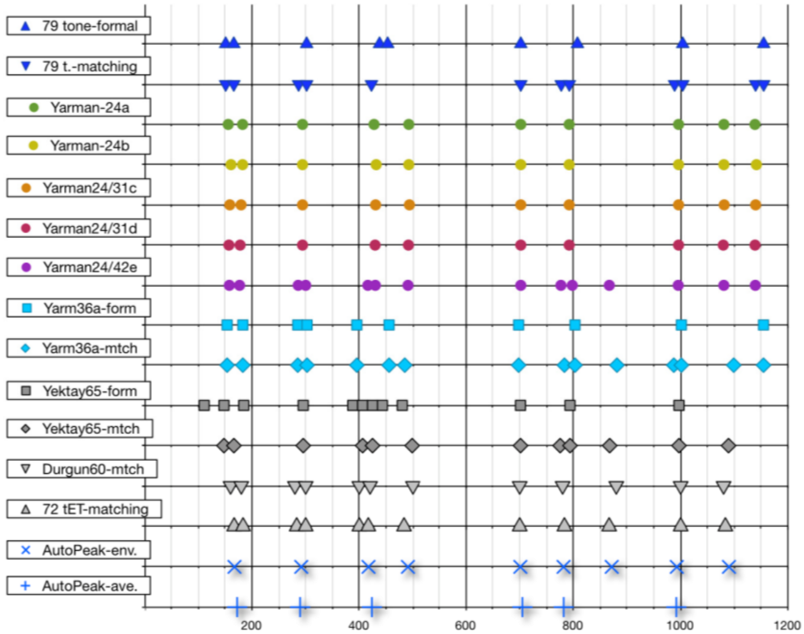
41 tET-matching	**WM:** 12.25	**WD:** 6.2	**WEM:** 14.97	**WED:** 7.57
34 tET-matching	**WM:** 9.67	**WD:** 5.36	**WEM:** **10.75**	**WED:** 5.96
29 tET-matching	**WM:** 12.12	**WD:** 4.1	**WEM:** 12.12	**WED:** **4.1**

THT 14 Mismatches in cents of the scale tones of various tunings with *SABA* auto-peaks (11 collated histograms) and the resultant *efficiency* (E) and *complexity* (C) percentages. "M" denotes the maximum difference and "D" denotes the mean of differences, with subscript "e" delineating values in reference to *Auto-Peak envelope* datapoints and subscript "a" delineating values in reference to *Auto-Peak average* datapoints, where best values are highlighted and in bold (while red, blue and purple text colors are cosmetic). Higher E and lower C are better.



31.99	13.79	41.13	17.73
7.34	**4.03**	12.59	6.91
10.95	6.13	13.69	7.66
12.25	6.39	15.31	7.98
12.58	6.63	15.73	8.29
12.1	6.43	15.12	8.04
8.95	4.87	13.93	7.57
28.3	13.75	40.43	19.64
26.92	9.69	44.87	16.15
18.14	8.9	31.1	15.27
8.66	5.17	11.55	6.9
11.6	7.25	15.46	9.67
9.85	5.88	13.13	7.84
WM	**WD**	**WEM**	**WED**

Continued table: Complexity and Efficiency scaled means of M's & D's



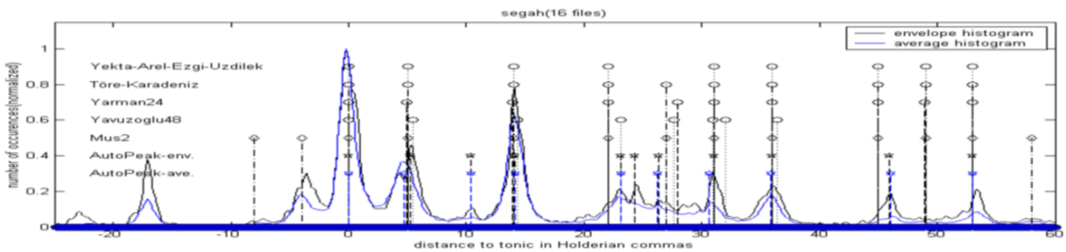
FHT 12 Plotting the data of THT 14 in comparison with the relevant histogram graphic in [Bozkurt et al., 2009].

SEGAH	Distance to tonic in cents															M_e	D_e	M_a	D_a	E	C
79 tone-formal	105.6			301.8		498			603.7		702	792.5	807.6	1003.8	1094.4	35.4	15.2	37.9	14.44	77.8	91.1
79 t.-matching	105.6	120.7	241.5	301.8	316.9	513.1	528.2	543.3	588.6	603.7	702	807.6	822.7	1034	1049.1	7.6	4.97	**7.8**	5.05	60	88.6
Yarman-24a	111.7		245		315.6		519.6			609.8	702		813.7	1017.6	1105.9	21.6	7.77	24.1	8.94	88.9	**66.7**
Yarman-24b	111.7		249		315.6		519.6			609.8	702		813.7	1017.6	1105.9	21.6	8.27	24.1	9.44	88.9	**66.7**
Yarman24/31c		114.5	250.6		313.8		522.3			612.5	702		816.5	1020.4	1041.2	16.8	6.18	17.7	7.61	88.9	74.2
Yarman24/31d		116.3	251.6		313.7		524.2			614.4	702		818.3	1022.2	1042.5	18.7	6.87	19.6	8.49	88.9	74.2
Yarman24/42e	109.3	123.2	239.1		313.9		525.2		600		704.2		818.6	1023.2	1043.8	6.5	4.27	9.3	4.83	80	81
Yarm36a-form		119.7		301.8		500.7				620.2	698.4		818.4	1017.1		24.5	13.86	25.4	16.13	**100**	80.6
Yarm36a-mtch	102.3	119.7	212.5	301.8	318.1	500.7	515.1	569	600.4		698.4		818.4	1017.1		22.1	9.69	24.6	9.2	75	75.0
Yektay65-form	110.8				313.8	498.5	516.9			609.2	701.5		812.3	1015.4	1107.7	23.8	8.57	26.3	9.07	77.8	89.2
Yektay65-mtch	110.8		240		313.8	498.5	516.9	553.8	590.8	609.2	701.5		812.3	1033.8		7.6	4.8	7.9	5.13	81.8	86.2
Durgun60-mtch	100	120	240		320	500	520	540	600		700		820	1040		9.7	3.79	**7.8**	**4.56**	81.8	85
72 tET-matching	100	116.7	233.3		316.7	500	516.7	550	600		700		816.7	1033.3	1050	**5.9**	**3.08**	8.3	4.76	75	87.5
41 tET-matching		117.1	234.1		322	497.6	526.8	556.1	585.4	614.6	702.4		819.5	1024.4		14.8	4.94	17.3	7.47	81.8	78
34 tET-matching	105.9		247.1		317.6	494.1	529.4	564.7	600		705.9		811.8		1058.8	19.6	8.86	17.1	6.83	90	73.5
29 tET-matching		124.1	248.3		331	496.6	537.9	537.9	579.3	620.7	703.4		827.6	1034.5		16.4	10.37	18.1	13.28	90	69
AutoPeak-env.	118.4		233.9	320.4		522.6		549.7	595.7		702.6	814		1039.2							
AutoPeak-ave.	106		233.7	318.8		523.2			594.8		694.9	812.2		1041.7							

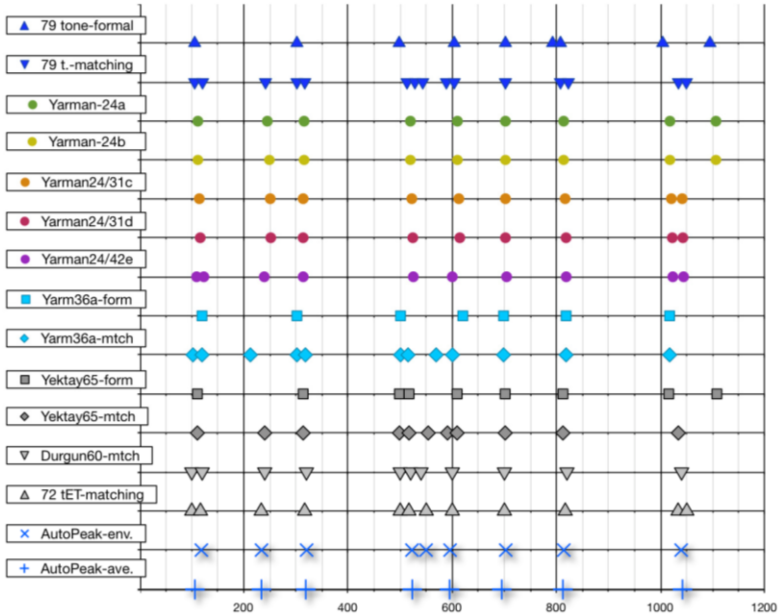
Continued table: Complexity-weighted (WM, WD) and Efficiency-scaled (WEM, WED) means of M's & D's

41 tET-matching	**WM:**	17.88	**WD:**	6.92	**WEM:**	21.85	**WED:**	8.45
34 tET-matching	**WM:**	19.61	**WD:**	8.38	**WEM:**	21.79	**WED:**	9.31
29 tET-matching	**WM:**	17.65	**WD:**	12.09	**WEM:**	19.61	**WED:**	13.44

THT 15 Mismatches in cents of the scale tones of various tunings with *SEGAH* auto-peaks (16 collated histograms) and the resultant *efficiency* (E) and *complexity* (C) percentages. "M" denotes the maximum difference and "D" denotes the mean of differences, with subscript "e" delineating values in reference to *Auto-Peak envelope* datapoints and subscript "a" delineating values in reference to *Auto-Peak average* datapoints, where best values are highlighted and in bold (while red, blue and purple text colors are cosmetic). Higher E and lower C are better.



45.62	18.45	58.65	23.72
9.39	6.11	15.65	10.18
22.85	8.36	25.71	9.4
22.85	8.86	25.71	9.96
18.55	7.41	20.87	8.34
20.59	8.26	23.17	9.29
9.03	5.2	**11.29**	6.5
28.42	17.08	28.42	17.08
25.3	10.23	33.73	13.64
30.7	10.81	39.47	13.9
9.26	5.93	11.32	7.25
10.35	4.94	12.66	**6.04**
8.58	**4.74**	11.44	6.32
WM	WD	WEM	WED
Continued table: Complexity and Efficiency scaled means of M's & D's			



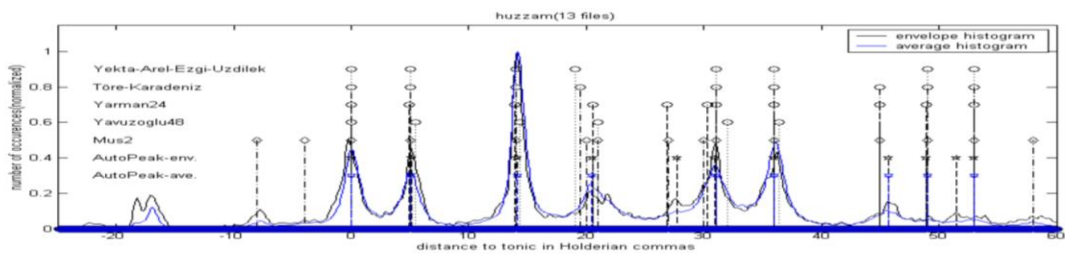
FHT 13 Plotting the data of THT 15 in comparison with the relevant histogram graphic in [Bozkurt et al., 2009].

HÜZZAM	Distance to tonic in cents															M_e	D_e	M_a	D_a	E	C
79 tone-formal	120.7		316.9			483	498		702	822.7		1018.9		1109.4		19.5	7.91	20	7.8	87.5	91.1
79 t.-matching	120.7	135.8	316.9	332.0		467.9	483	633.9	702	822.7	837.8	1018.9	1034	1109.4	1169.8	8.1	3.94	7.7	3.8	64.3	88.6
Yarman-24a	111.7		315.6			466.3		609.7	702	813.7		1017.6		1105.9	1173.2	18.6	6.79	18.6	6.32	**100**	**62.5**
Yarman-24b	111.7		315.6			468.6		609.8	702	813.7		1017.6		1105.9	1156.6	18.5	7.08	18.5	6.6	**100**	**62.5**
Yarman24/31c	114.5		313.8			469.5		612.5	702	816.5		1020.4	1041.2	1108.9	1164.8	15.8	4.39	15.8	4.97	90	71
Yarman24/31d	116.3		313.7			469.9		614.4	702	818.3		1022.2	1042.5	1110.1	1165.3	13.9	4.63	13.9	5.54	90	71
Yarman24/42e	109.3	123.2	313.9			443.8	471.6	621.3	704.2	818.6		1023.2	1043.8	1117.1	1167	8.8	6.07	10	6.26	75	78.6
Yarm36a-form	119.7		301.8			471.4			698.4	818.4		1017.1				18.5	8.92	19	9.37	**100**	83.3
Yarm36a-mtch	119.7		301.8	318.1		471.4		620.2	698.4	818.4		1017.1	1068.9	1100.3	1170.7	18.5	6.54	16.7	7.09	81.8	75
Yektay65-form	110.8		313.8	406.2	424.6	443.1	461.5		701.5	812.3		1015.4		1107.7		20.2	5.37	18.4	4.89	70	89.2
Yektay65-mtch	110.8		313.8			443.1	461.5	627.7	701.5	812.3		1015.4	1033.8	1107.7	1163.1	**5.9**	**2.42**	**7**	**2.05**	81.8	86.2
Durgun60-mtch	120		320			460		620	700	820		1020	1040	1100	1160	8.3	4.42	9	4.96	90	85
72 tET-matching	116.7		316.7			466.7		633.3	700	816.7		1033.3		1116.7	1166.7	8.4	2.98	7.7	3.35	**100**	87.5
41 tET-matching	117.1		322			468.3		614.6	702.4	819.5		1024.4		1112.2	1170.7	13.7	5.6	13.7	5.22	**100**	78
34 tET-matching	105.9		317.6			458.8		635.3	705.9	811.8		1023.5	1058.8	1094.1	1164.7	14.2	6.58	14.9	6.92	90	73.5
29 tET-matching		124.1		331		455.2		620.7	703.4		827.6	1034.5		1117.2	1158.6	13	7.63	11.8	7.45	**100**	69
AutoPeak-env.	115.7		319.7			463.5		628.3	699.8	814.6		1035.6		1108.3	1165.1						
AutoPeak-ave.	113		320.8			463		628.3	701.2	815.8		1033.8		1109							

Continued table: Complexity-weighted (WM, WD) and Efficiency-scaled (WEM, WED) means of M's & D's

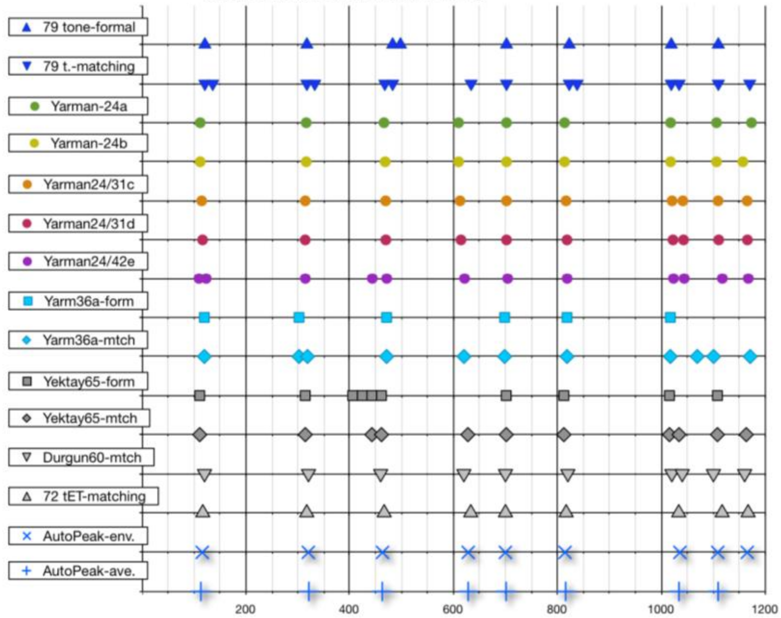
41 tET-matching	**WM:** 15.83	**WD:** 6.25	**WEM:** 15.83	**WED:** 6.25
34 tET-matching	**WM:** 16.15	**WD:** 7.5	**WEM:** 17.95	**WED:** 8.33
29 tET-matching	**WM:** 13.2	**WD:** 8.03	**WEM:** 13.2	**WED:** 8.03

THT 16 Mismatches in cents of the scale tones of various tunings with *HÜZZAM* auto-peaks (13 collated histograms) and the resultant *efficiency* (E) and *complexity* (C) percentages. "M" denotes the maximum difference and "D" denotes the mean of differences, with subscript "e" delineating values in reference to *Auto-Peak envelope* datapoints and subscript "a" delineating values in reference to *Auto-Peak average* datapoints, where best values are highlighted and in bold (while red, blue and purple text colors are cosmetic). Higher E and lower C are better.



25.41	10.11	29.04	11.55
9.96	4.88	15.5	7.6
18.6	6.56	18.6	6.56
18.5	6.84	18.5	6.84
17.14	5.08	19.04	5.64
15.08	5.52	16.75	6.13
10.91	7.16	14.55	9.54
22.66	11.05	22.66	11.05
19.8	7.67	24.2	9.37
24.46	6.5	34.94	9.28
7.98	**2.77**	**9.75**	**3.38**
10.6	5.75	11.77	6.39
10.06	3.95	10.06	3.95
WM	WD	WEM	WED

Continued table: Complexity and Efficiency scaled means of M's & D's



FHT 14 Plotting the data of THT 16 comparison with the relevant histogram graphic in [Bozkurt et al., 2009].

PANORAMA A	M_{em}	D_{em}	M_{am}	D_{am}	E_m	C_m	M_m	D_m	M_c	D_c	GWM	GWD
79 tone-formal (79 per 159 tET)	23.1	10	24.6	10.5	87.1	91.1	23.9	10.2	29.4	12.6	29.5	12.6
79 tone-matching	**7.1**	**3.6**	**7.4**	**3.9**	64.6	89.2	**7.3**	**3.7**	**8.8**	**4.5**	**8.9**	**4.6**
Yarman-24a	14.8	6.4	19.7	7.4	**92.7**	**68.1**	17.2	6.9	17.2	6.9	17.4	7
Yarman-24b	16.2	6.5	21.1	7.8	**92.7**	**68.1**	18.7	7.2	18.7	7.2	18.9	7.2
Yarman24/31c	15.7	6	20	6.8	85.8	75.3	17.9	6.4	19.1	6.9	19.3	7
Yarman24/31d	15.9	6.3	19.9	7	85.8	75.3	17.9	6.7	19.2	7.1	19.3	7.2
Yarman24/42e	11.3	6	12.7	6.2	79.2	81.5	12	6.1	13.7	6.9	13.8	7
Yarman36a-formal	19.6	10.4	20.3	10.3	**92**	82.4	19.9	10.4	22.8	11.8	23	11.9
Yarman36a-matching	18.2	7.7	18.9	7.9	77.6	75.9	18.5	7.8	20	8.4	20.2	8.5
Yektay65-formal (in 65 tET)	18.5	7.8	19.8	7.6	72.4	89.1	19.2	7.7	23.2	9.3	23.3	9.3
Yektay65-matching (in 65 tET)	9.1	4	9	4.7	79.4	86.5	9.1	4.3	10.7	5.1	10.8	5.2
Durgun60-matching (in 60 tET)	9.4	4.6	9.3	4.9	80.8	85.4	9.4	4.7	11	5.5	11.1	5.6
72 tET-matching	8.7	3.9	9.1	4.6	82.5	87.8	8.9	4.3	10.7	5.1	10.7	5.1
41 tET-matching	13.1	6.4	13	6.1	87.2	79.1	13	6.3	14.5	7	14.6	7
34 tET-matching	14.6	7	12.9	5.8	**90**	75.2	13.7	6.4	14.7	6.9	14.8	7
29 tET-matching	15.8	7.3	15.9	7.8	**93.4**	70.5	15.8	7.5	16.2	7.7	16.3	7.8
YAEU (24 per 53-tET)	22.5	8	23.3	9.1	**98.4**	74.1	22.9	8.6	24.3	9.1	24.4	9.1
Mus2 (53 tET-formal)	16.9	7.3	16.7	7.3	73.5	85.5	16.8	7.3	19.7	8.6	19.8	8.6
Töre-Karadeniz (41 per 106 tET)	23	8.7	28.8	11.6	**95.8**	83.5	25.9	10.2	29.9	11.7	29.9	11.7
Old Yarman-24a	18.4	7	19.4	7.2	**92.7**	**68.1**	18.9	7.1	18.9	7.1	18.9	7.1
Yavuzoğlu-48 (in 48 tET)	23.9	11.2	23.7	12.2	**95.6**	86.3	23.8	11.7	28.2	13.8	28.3	13.9

THT 17 Grand averages of all mismatches in cents of the scale tones of various tunings with auto-peaks, and the resultant mean *efficiency* (E) and mean *complexity* (C) percentages (constituting altogether PANORAMA A – with the bottom 5 rows drawn from [Bozkurt et al., 2009]). "M" denotes the maximum difference and "D" denotes the average of differences; with subscript "em" delineating mean values in reference to *Auto-Peak envelope* datapoints, subscript "am" delineating mean values in reference to *Auto-Peak average* datapoints, subscript "m" delineating the average of em's and am's as well as all preceding E's and C's, and subscript "c" delineating the complexity scaling (similar to for GWM: *Grand average of complexity-weighted maximums* and GWD: *Grand average of complexity-weighted average of differences* pulled out of continued THT 8 to THT 16), where best values are highlighted and in bold (while red, blue, purple, green and turquoise text colors are cosmetic). As it so happens, 79 MOS 159-tET (the peak-matching version) outclasses its competition in this panorama even after complexity upscaling under the rightmost four columns. Higher E and lower C are better, while smaller deviations in all other measures are always better.

PANORAMA B (Bulk vs. Peak-matching Performance)	Pre-normalized upscaled c				Normalized to percentages				Average of performance maximums	Average of performance mean diffs.
	CEM Perf.	CED Perf.	GEM Perf.	GED Perf.	CEM Perf. %	CED Perf. %	GEM Perf. %	GED Perf. %		
79 tone-formal (79 per 159 tET)	33.75	14.44	33.92	14.50	38%	43%	38%	43%	38%	43%
79 tone-matching	**13.61**	**7.02**	**13.7**	**7.07**	**95%**	**88%**	**95%**	**88%**	**95%**	**88%**
Yarman-24a	18.61	7.43	18.78	7.5	70%	83%	69%	83%	69%	83%
Yarman-24b	20.17	7.71	20.36	7.78	64%	80%	64%	80%	64%	80%
Yarman24/31c	22.31	8.04	22.5	8.11	58%	77%	58%	77%	58%	77%
Yarman24/31d	22.36	8.32	22.55	8.4	58%	74%	58%	74%	58%	74%
Yarman24/42e	17.25	8.71	17.42	8.8	75%	71%	75%	71%	75%	71%
Yarman36a-formal	24.77	12.86	24.96	12.97	52%	48%	52%	48%	52%	48%
Yarman36a-matching	25.77	10.86	25.97	10.94	50%	57%	50%	57%	50%	57%
Yektay65-formal (in 65 tET)	31.99	12.79	32.11	12.84	40%	48%	41%	48%	40%	48%
Yektay65-matching (in 65 tET)	**13.51**	**6.46**	**13.58**	**6.5**	**96%**	**96%**	**96%**	**96%**	**96%**	**96%**
Durgun60-matching (in 60 tET)	**13.61**	**6.86**	**13.69**	**6.92**	**95%**	**90%**	**95%**	**90%**	**95%**	**90%**
72 tET-matching	**12.94**	**6.18**	**13.02**	**6.22**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
41 tET-matching	16.6	8.01	16.7	8.08	78%	77%	78%	77%	78%	77%
34 tET-matching	16.35	7.66	16.5	7.73	79%	81%	79%	80%	79%	81%
29 tET-matching	17.38	8.26	17.48	8.34	74%	75%	74%	75%	74%	75%
YAEU (24 per 53-tET)	24.67	9.22	24.79	9.21	52%	67%	53%	68%	52%	67%
Mus2 (53 tET-formal)	26.81	11.65	26.88	11.67	48%	53%	48%	53%	48%	53%
Töre-Karadeniz (41 per 106 tET)	31.19	12.23	31.15	12.21	41%	51%	42%	51%	42%	51%
Old Yarman-24a	20.38	7.65	20.38	7.65	64%	81%	64%	81%	64%	81%
Yavuzoğlu-48 (in 48 tET)	29.45	14.45	29.58	14.51	44%	43%	44%	43%	44%	43%

THT 18 Efficiency-upscaled M_c, D_c and GWM, GWD; designated as CEM, CED and GEM, GED respectively – with resultant c values converted to percentages via taking as basis the smallest numbers each in the first four columns under the "Pre-normalized" heading. Please note that CEM & CED are the **C**omplexity-weighed AND **E**fficiency-upscaled **M**aximums (or the mean of the average of **D**ifferences), while GEM & GED are the **G**rand-averaged version of complexity-weighed AND **E**fficiency-upscaled Maximums (or the mean of the average of **D**ifferences), with smaller values being always better. The operation uses the formula % = 100 x [(*smallest among the list of* [*complexity-weighted c / corresponding efficiencies*]) / (*complexity-weighted c / current efficiency*)]. Rightmost two columns feature the average of CEM, GEM & the average of CED, GED correspondingly. In this "*bulk vs. peak-matching performance*" panorama that I dub PANORAMA B – with the bottom 5 rows drawn from [Bozkurt et al., 2009] – all inefficient players are aggressively taken down to the effect that 72-tET comes out on top, followed by 65-tET, 60-tET and 79 MOS 159-tET (*e.g.*, the peak-matching version). Best values are highlighted and in bold, while worst values are striped in red.

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